Quantum Technology at CERN

Leonardo Lavagna

THE BIG STORY

The Quantum Apocalypse Is Coming. Be Very Afraid

What happens when quantum computers can finally crack encryption and break into the world's best-kept secrets? It's called Q-Day—the worst holiday maybe ever.

AMIT KATWALA

MAR 24, 2025 6:00 AM

Meet Willow, our state-of-the-art quantum chip

Dec 09, 2024
9 min read

Our new chip demonstrates error correction and performance that paves the way to a useful, large-scale quantum computer

Last updated: June 12, 2025

Today I'm delighted to announce Willow, our latest quantum chip. Willow has state-of-the-art performance across a number of metrics, enabling two major achievements.

- The first is that Willow can reduce errors exponentially as we scale up using *more* qubits. This cracks a key challenge in quantum error correction that the field has pursued for almost 30 years.
- Second, Willow performed a standard benchmark computation in under five minutes that would take one of today's fastest supercomputers 10 septillion (that is, 10^{25}) years — a number that vastly exceeds the age of the Universe.

called G-Day—the worst holiday maybe ever.

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Google unveils 'mindboggling' quantum computing chip

Chip takes minutes to complete tasks that would otherwise take 10,000,000,000,000,000,000,000,000 years

Dr Erik Lucero, lead engineer of Google Quantum AI, points to motherboards while leading media on a tour of the Quantum Computing Lab in September 2022. Photograph: Frederic J Brown/AFP/Getty Images

NEWSLETTERS SI

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Dec 09, 2024
9 min read

Last up

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Google unveils 'mindboggling' quantum computing chip

NEWSLETTERS

SECTIONS

NEW YORK POST

Science Space Environment Wildlife Archaeology

DEAL DROP 101+ best Memorial Day 2026 sales and deals we're tracking so far

SCIENCE

Google claims quantum chip may prove existence of parallel universes

By Ben Cost

Published Dec. 15, 2024, 4:58 p.m. ET

media on a tour of the Quantum Computing Lab in September 2022. Photograph: Frederic J Brown/AFP/Getty Images


Human Genome Project
Article
Talk
From Wikipedia, the free encyclopedia


www.smbc-comics.com/comic/the-talk-3


"THE TALK"
BY SCOTT AARONSON & ZACH WEINERSMITH
WHAT ARE YOU READING?!
Popular Scientism
Quantum
NOTHING! NOTHING!
YOU'RE GROWING UP SO FAST. I... I THINK IT'S TIME YOU AND I HAD... "THE TALK"
THE QUANTUM COMPUTING TALK.

- Overview
- Interactive Discussion on Quantum Computing and Algorithms

Noisy intermediate-scale quantum computing

From Wikipedia, the free encyclopedia

(Redirected from Noisy intermediate-scale quantum era)

Noisy intermediate-scale quantum (**NISQ**) **computing** is characterized by quantum processors containing up to 1,000 qubits which are not advanced enough yet for fault-tolerance or large enough to achieve quantum advantage.[1][2] These processors, which are sensitive to their environment (noisy) and prone to quantum decoherence, are not yet capable of continuous quantum error correction. This intermediate-scale is defined by the quantum volume, which is based on a moderate number of qubits and gate fidelity. The **NISQ era** is the current state of quantum computer technology,[3][4][5] and the term was coined by John Preskill in 2018.[6][4]

According to Microsoft Azure Quantum's scheme, NISQ computation is considered level 1, the lowest of the quantum computing implementation levels.[7][8]

In October 2023, the 1,000 qubit mark was passed for the first time by Atom Computing's 1,180 qubit quantum processor.[9] However, as of 2024, only two quantum processors have over 1,000 qubits, with sub-1,000 quantum processors still remaining the norm.[10]

Noisy intermediate-scale quantum computing

7 languages

Article Talk

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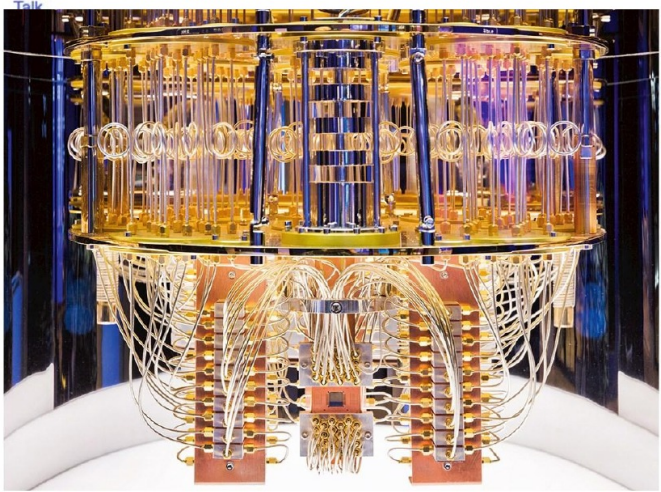

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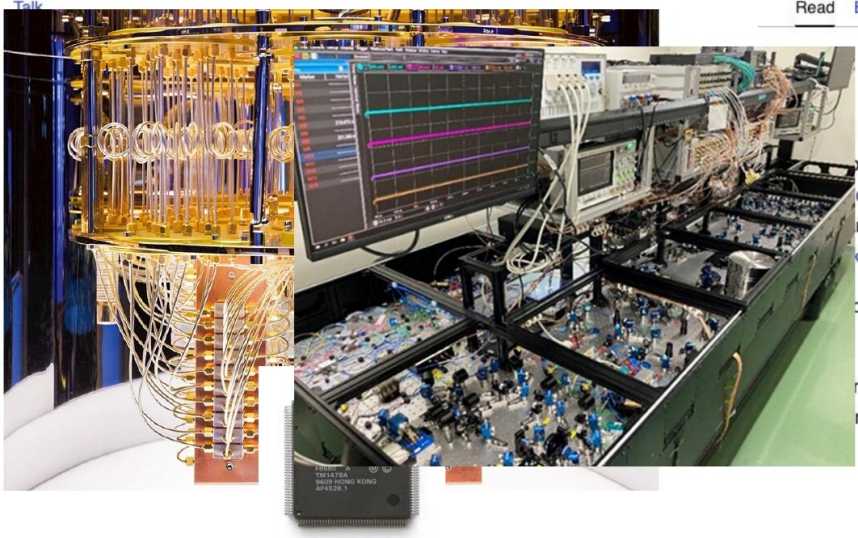

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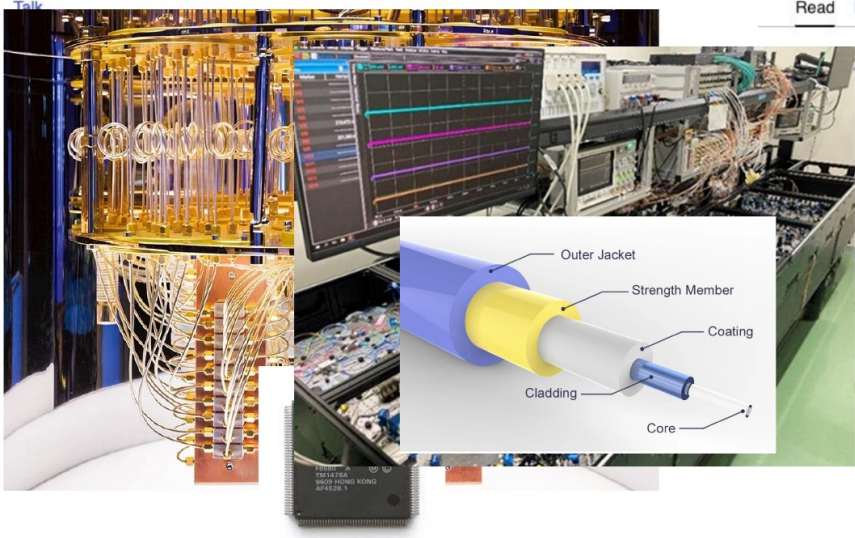

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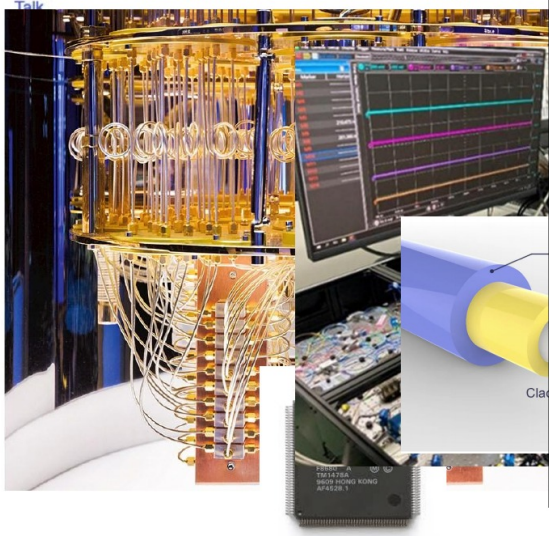

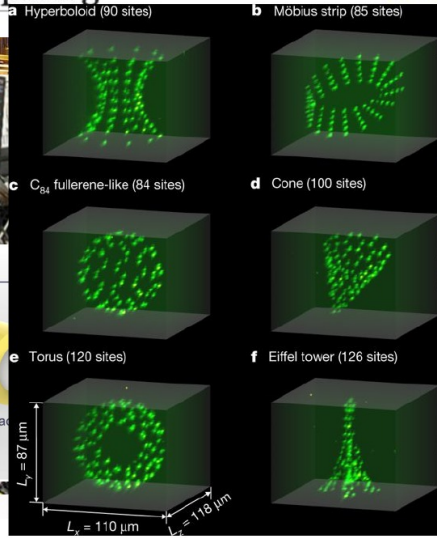

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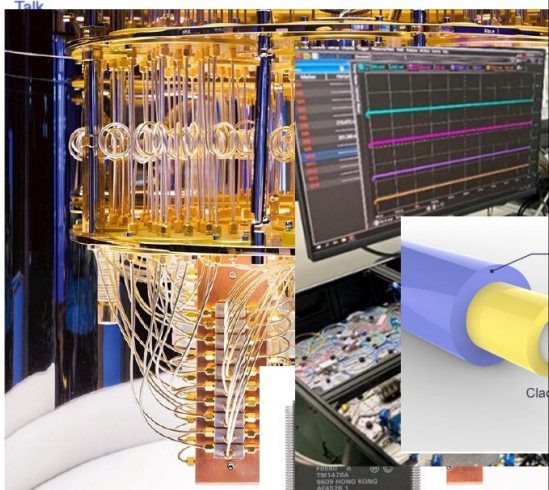

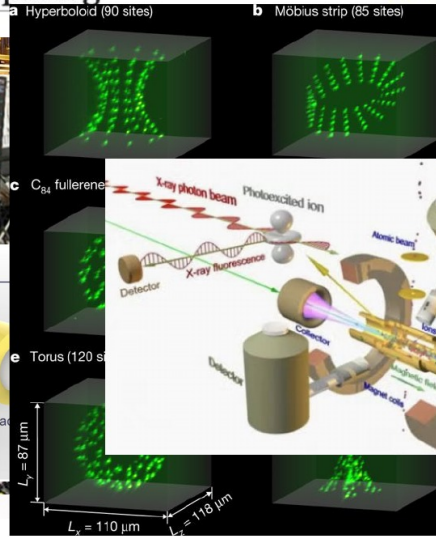

Quantum Physics

[Submitted on 10 Feb 2025]

A Review and Collection of Metrics and Benchmarks for Quantum Computers: definitions, methodologies and software

Deep Lall, Abhishek Agarwal, Weixi Zhang, Lachlan Lindoy, Tobias Lindström, Stephanie Webster, Simon Hall, Nicholas Chancellor, Petros Wallden, Raul Garcia-Patron, Elham Kashefi, Viv Kendon, Jonathan Pritchard, Alessandro Rossi, Animesh Datta, Theodoros Kapourniotis, Konstantinos Georgopoulos, Ivan Rungger

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How to Build a Quantum Supercomputer: Scaling from Hundreds to Millions of Qubits

Masoud Mohseni, Artur Scherer, K. Grace Johnson, Oded Wertheim, Matthew Otten, Namit Anand, Navid Anjum Aadit, Yuri Alexeev, Gilad Ben-Shach, Kirk M. Bresniker, Kerem Y. Camsari, Barbara Chapman, Soumitra Chatterjee, Shuvro Chowdhury, Gebremedhin A. Dagnew, Tom Dvir, Aniello Esposito, Farah Fahim, Michael Ferguson, Marco Fiorentino, Archit Gajjar, Katerina Gratsea, Gaurav Gyawali, Christian Heiter, Ali H. Z. Kavaki, Abdullah Khalid, Xiangzhou Kong, Bohdan Kulchytskyy, Elica Kyoseva, Ruoyu Li, P. Aaron Lott, Igor L. Markov, Robert F. McDermott, Lucas Morais, Giacomo Pedretti, Pooja Rao, Eleanor Rieffel, Allyson Silva, John Sorebo, Panagiotis Spentzouris, Ziv Steiner, Boyan Torosov, Davide Venturelli, Robert J. Visser, Zak Webb, Xin Zhan, Yonatan Cohen, Pooya Ronagh, Alan Ho, Raymond G. Beausoleil, John M. Martinis

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[Submitted on 10 Feb 2025]

Quantum Physics

[Submitted on 12 Feb 2026 (v1), last revised 5 May 2026 (this version, v2)]

The Pinnacle Architecture: Reducing the cost of breaking RSA-2048 to 100 000 physical qubits using quantum LDPC codes

Paul Webster, Lucas Berent, Omprakash Chandra, Evan T. Hockings, Nouédyn Baspin, Felix Thomsen, Samuel C. Smith, Lawrence Z. Cohen
Allyson
Ronagh, Alan
Microsoft

[Submitted on 13 Mar 2026 (this version, v3)]

Quantum Supercomputer: Scaling from Hundreds to Millions of Qubits

Johnson, Oded Wertheim, Matthew Otten, Namit Anand, Navid Anjum Aadit, Yuri Alexeev, Gilad Ben-Shach, Kirk M.
Soumitra Chatterjee, Shuvro Chowdhury, Gebremedhin A. Dagnew, Tom Dvir, Aniello Esposito, Farah Fahim,
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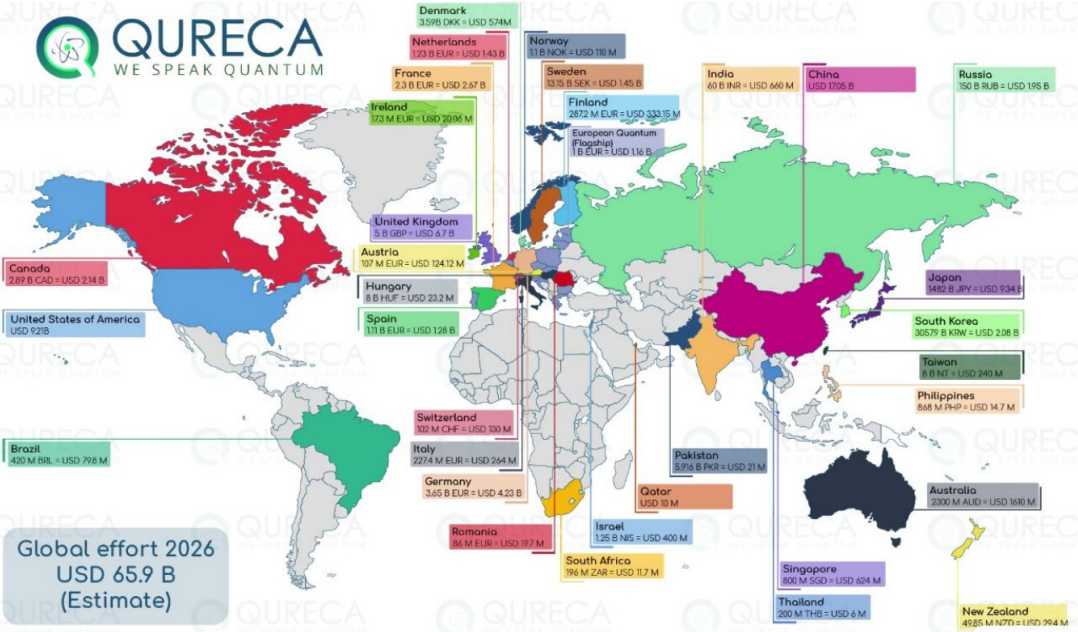

Global effort 2026
USD 65.9 B
(Estimate)


Versailles
Parigi
Strasburgo
Stoccarda
Slovacchia
Košice
Rennes
Le Mans
Linz
Vienna
Monaco di Baviera
Friburgo in Brisgovia
Salisburgo
Austria
Budapest
Debrecen
Angers
Tours
Basilea
Zurigo
Innsbruck
Nantes
Digione
Graz
Ungheria
Francia
Svizzera
Losanna
Poitiers
La Rochelle
Ginevra
Seghedino
Arad
Slovenia
Clermont-Ferrand
Lione
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Trieste
Milano
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Venezia
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Grenoble
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Croazia
Bordeaux
Pola
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Београд
Parma
Bosnia ed Erzegovina
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Охрид
Olbia
Napoli
Sassari
Salerno
Matera


Versailles
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Bari
Sassari
Napoli
Salerno
Matera
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CERN
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ALICE
LHC – 27 km
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UNIVERSITÀ DI ROMA


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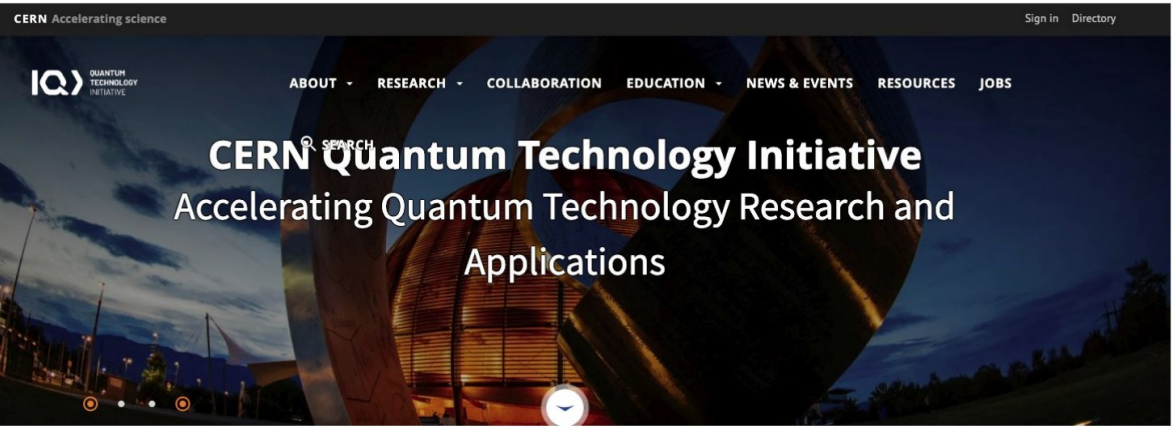
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- ResearchGate
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- Github
- HackerRank
- Kaggle
- Google Scholar

https://leonardolavagna.github.io/

“The Talk”

- **Overview**
- **Interactive Discussion on Quantum Computing and Algorithms**

Bits, Qubits, (Quantum) Gates, (Quantum) Turing Machines, and Public key cryptography

How many of you have done programming?

Bits, Qubits, (Quantum) Gates, (Quantum) Turing Machines, and Public key cryptography

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We will not need an actual “quantum” computer.

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What is information? Think of a Morse message.

Bits and bit strings

The basic unit of information is the bit: $\{0, 1\}$.

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We can represent "knowledge" as strings of bits:

$$x_1, x_2, \ldots, x_n \in \{0, 1\}.$$

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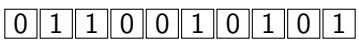

Exercise: how many strings?

If we have n bits, how many strings can we write?

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number of bits	strings	how many
1	$0, 1$	2
2	$00, 01, 10, 11$	2^2
3	$000, 001, 010, 011, 100, 101, 110, 111$	2^3
n	$x_1 \ldots x_n$	2^n

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- $2^{32} \simeq 4 \cdot 10^9$;

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One byte $= 8$ bits, $1\,\mathrm{kB} = 2^{10}$ bytes, $1\,\mathrm{MB} = 2^{20}$ bytes, $1\,\mathrm{GB} = 2^{30}$ bytes.

Encodings

Of course, with different encodings we could work with trits $\{0, 1, 2\}$, integers, and so on.

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Exercise: encode the word "LEO".

$$A = 00000, \quad B = 00001, \quad C = 00010, \quad D = 00011,$$
$$E = 00100, \quad \ldots, \quad L = 01011, \quad \ldots, \quad O = 01110.$$

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ASCII uses 8 bits per character.

Exercise: encode a 2×2 RGB image

A 2×2 image has four pixels. Each RGB channel can take values in $\{0, \ldots, 255\}$.

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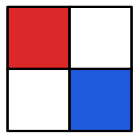

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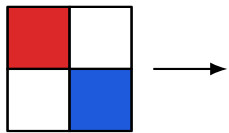

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With a computer we want to encode information but also to process it: classical computing.

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We manipulate information using combinations of gates transforming bit strings.

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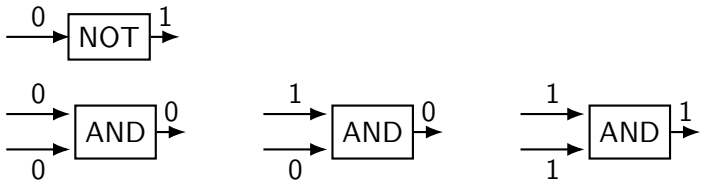

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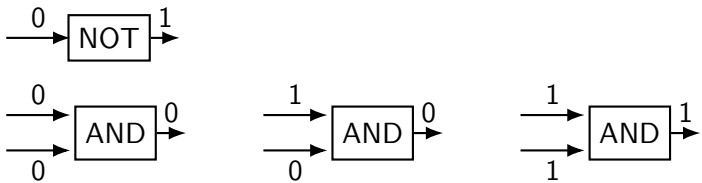

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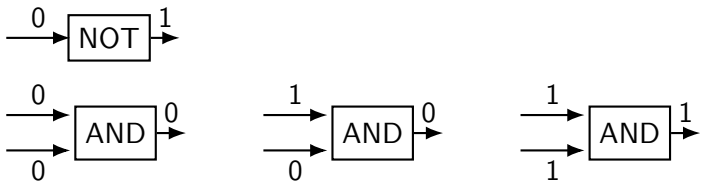

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These gates have tabular representations.

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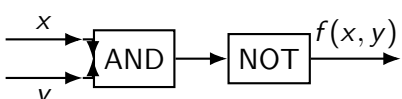

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Exercise: build a Boolean function that realizes a simple comparison test.

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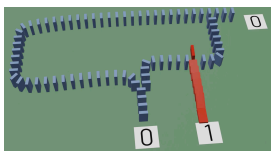

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Because any circuit still has to work with a fixed number of bits.

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It works for every m if it fits the memory.

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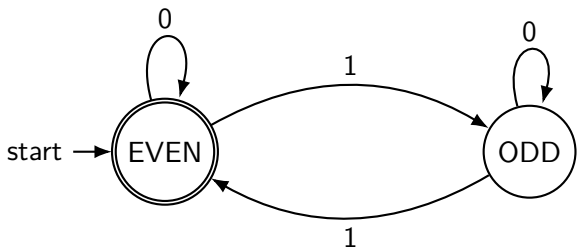

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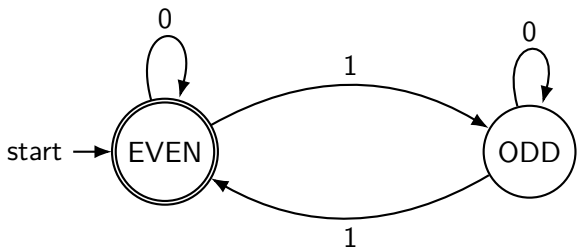

Exercise: design an automaton for strings with an odd number of ones.

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Exercise: define a regular expression for the set of strings with an even number of ones.

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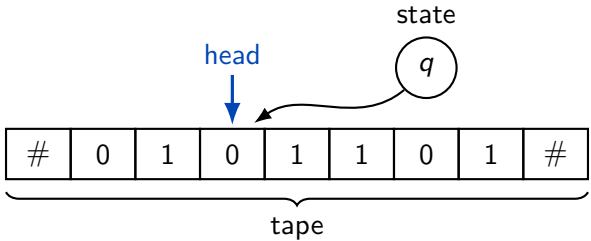
Turing machines



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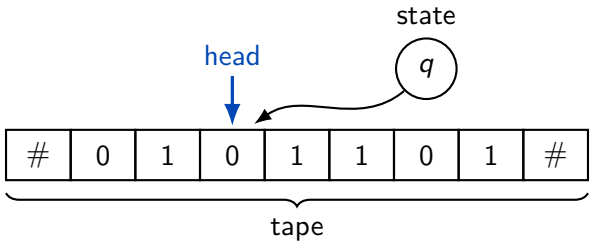

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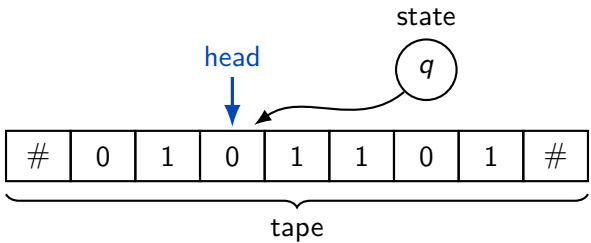

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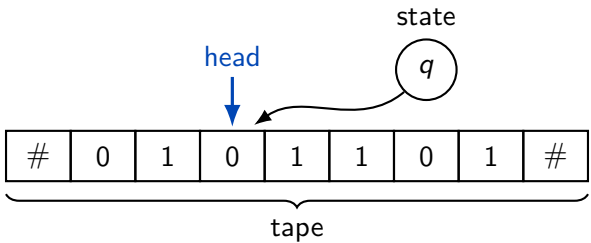

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Exercise: Turing machine for AND(x, y).

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Let us discuss the unit of quantum information and the relevant gates / automata and Turing machines.

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Of such differences, consider an asymmetric example common in peer-to-peer internet communications.

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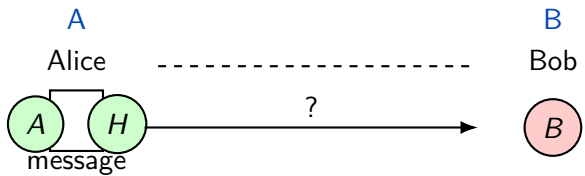

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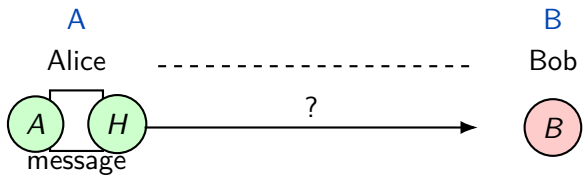

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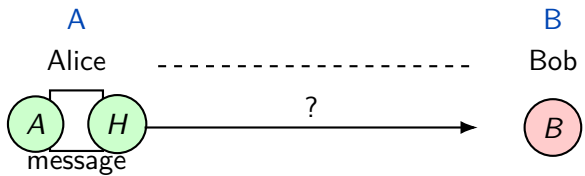

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The double-lock trick gives the idea.

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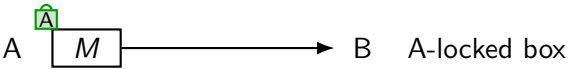

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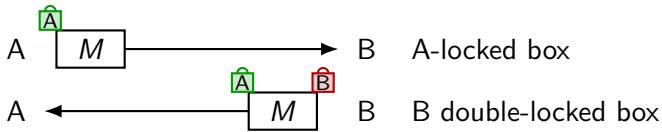

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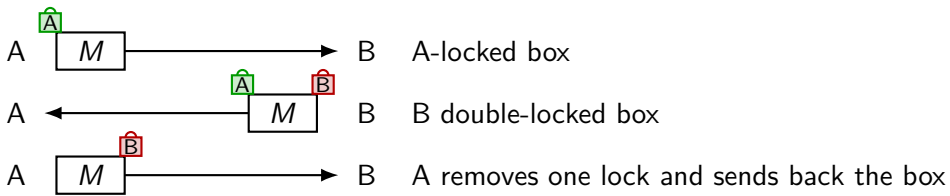

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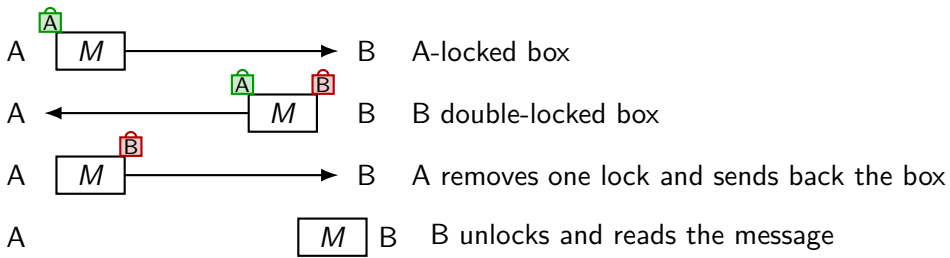

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In this arithmetic we can define a classically hard problem to solve with a Turing machine.

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$$n = p \cdot q$$

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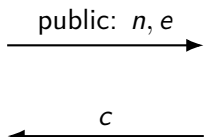

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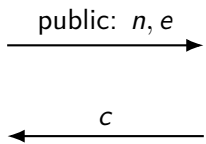

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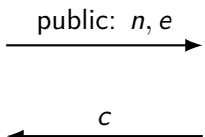

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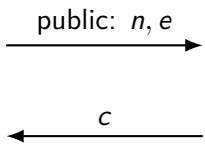
- decrypts $c^d \bmod n = M$.



B

- has message M;
- computes $c = M^e \bmod n$;

RSA: public and private numbers

A

- chooses p, q secret;
- $n = pq$;
- $\varphi(n) = (p-1)(q-1)$;
- chooses e with $\gcd(e, \varphi(n)) = 1$;
- finds d with $ed = 1 \bmod \varphi(n)$;
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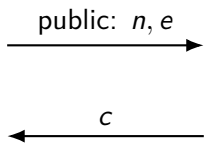

B

- has message M;
- computes $c = M^e \bmod n$;
- sends c to A.

RSA: public and private numbers continued

In practice, if $n = 4$, $p = 2$, $q = 2$, then

$$\varphi(n) = (2 - 1)(2 - 1) = 1.$$

RSA: public and private numbers continued

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If $n = 33$, then one can take $p = 3$, $q = 11$ and

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THE END