Hybrid Quantum-Classical Framework for Anomaly Detection in Time Series with QUBO formulation and QAOA

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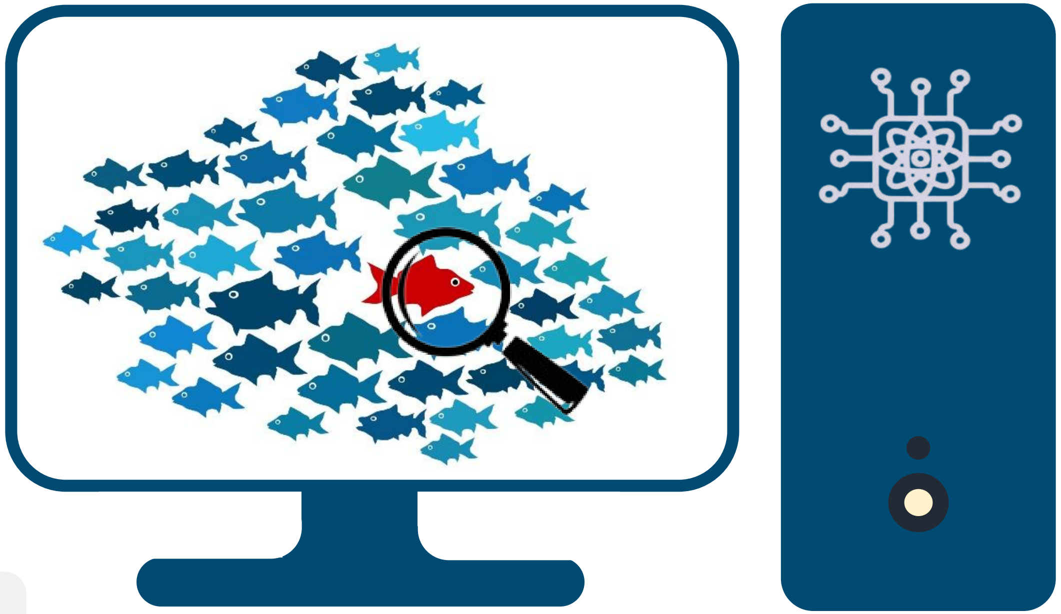
Motivation & Context

Quantum Computing

+

Anomaly Detection

Quantum Anomaly Detection



Motivation & Context

Why bother complicating things up with Quantum Computing?

Aren’t classical anomaly detection architectures good enough?

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Aren’t classical anomaly detection architectures good enough?

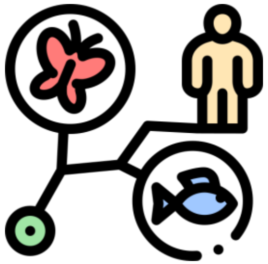
Motivation & Context

Use-case dependency:
Methods are often tailored to specific applications, reducing adaptability.

Limited sensitivity control:
Hard to adjust how strict or lenient the detection should be.

Interpretability:
Many models act as black boxes, making results hard to explain.

Evolving anomaly types:
New anomalies require frequent model updates to stay effective.



Zamanzadeh Darban, Zahra, et al. "Deep learning for time series anomaly detection: A survey." ACM Computing Surveys 57.1 (2024): 1-42.

A. B. Nassif, M. A. Talib, Q. Nasir, and F. M. Dakalbab, “Machine learning for anomaly detection: A systematic review,” Ieee Access, vol. 9,pp. 78 658–78 700, 2021

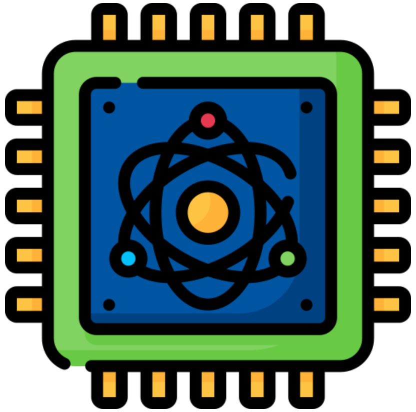
Shaukat, Kamran, et al. "A review of time-series anomaly detection techniques: A step to future perspectives." Advances in information and communication: proceedings of the 2021 future of information and communication conference (FICC), volume 1. Springer International Publishing, 2021.

The work at a glance

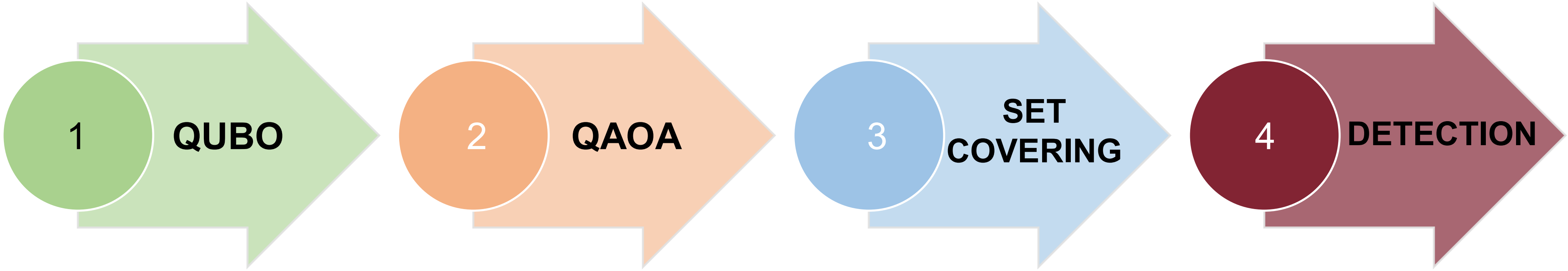
Focus
Time series analysis

Task
Anomaly Detection

Novelty
Quantum Optimization



Quantum Anomaly Detection Pipeline



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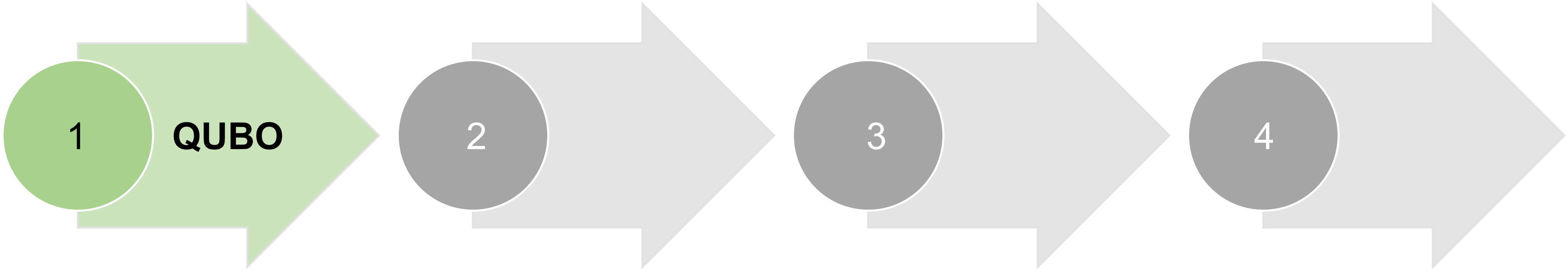


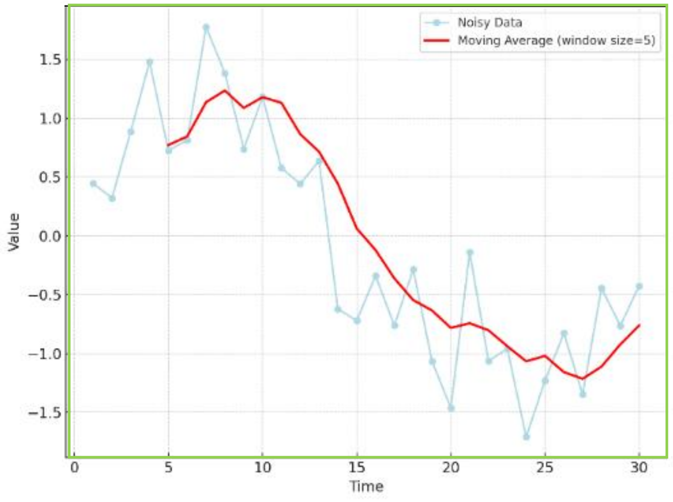
QUBO: Quadratic Unconstrained Binary Optimization

$$\min f(X) = C^T X + X^T Q X$$

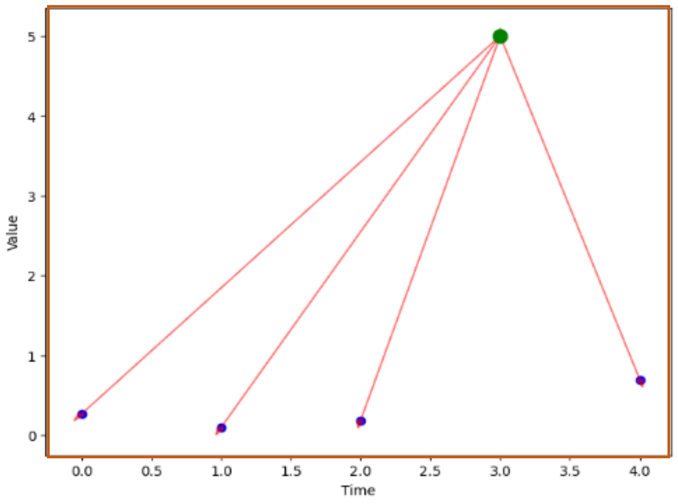
QUBO: Quadratic Unconstrained Binary Optimization

$$\min f(X) = \underbrace{C^T X}_{\text{Linear}} + \underbrace{X^T Q X}_{\text{Quadratic}}$$

Model



Distance



QUBO: Quadratic Unconstrained Binary Optimization

$$\min f(X) = C^T X + X^T Q X$$

$$\min f(X) = \begin{bmatrix} c_1 & \dots & c_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} x_1 & \dots & x_N \end{bmatrix} \begin{bmatrix} q_{1,1} & \cdots & q_{1,N} \\ \vdots & \ddots & \vdots \\ q_{N,1} & \cdots & q_{N,N} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

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$$x_i \in \{0,1\}$$

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$$f(X) = \alpha \sum_{i}^{N} c(x_i)\, x_i + \beta \sum_{(i,j), i \neq j}^{N} q_{i,j}(x_i, x_j)\, x_i x_j$$

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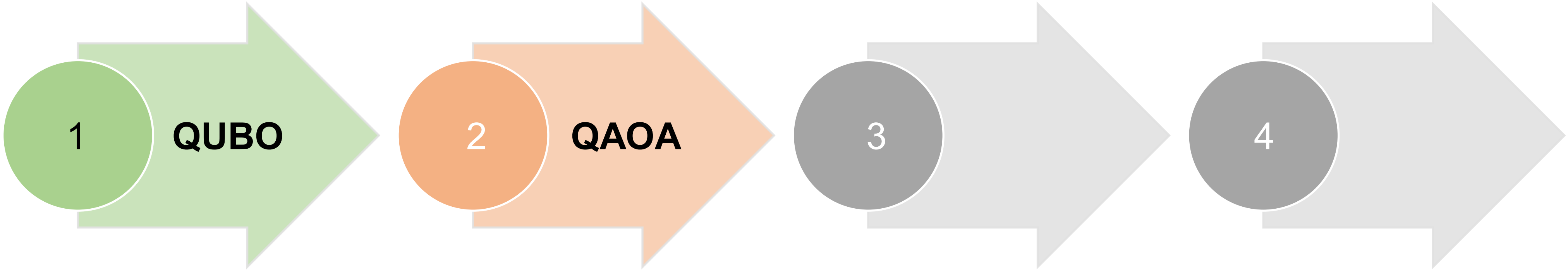
$\alpha \in (-1,0)$ $\beta \in (0,1)$

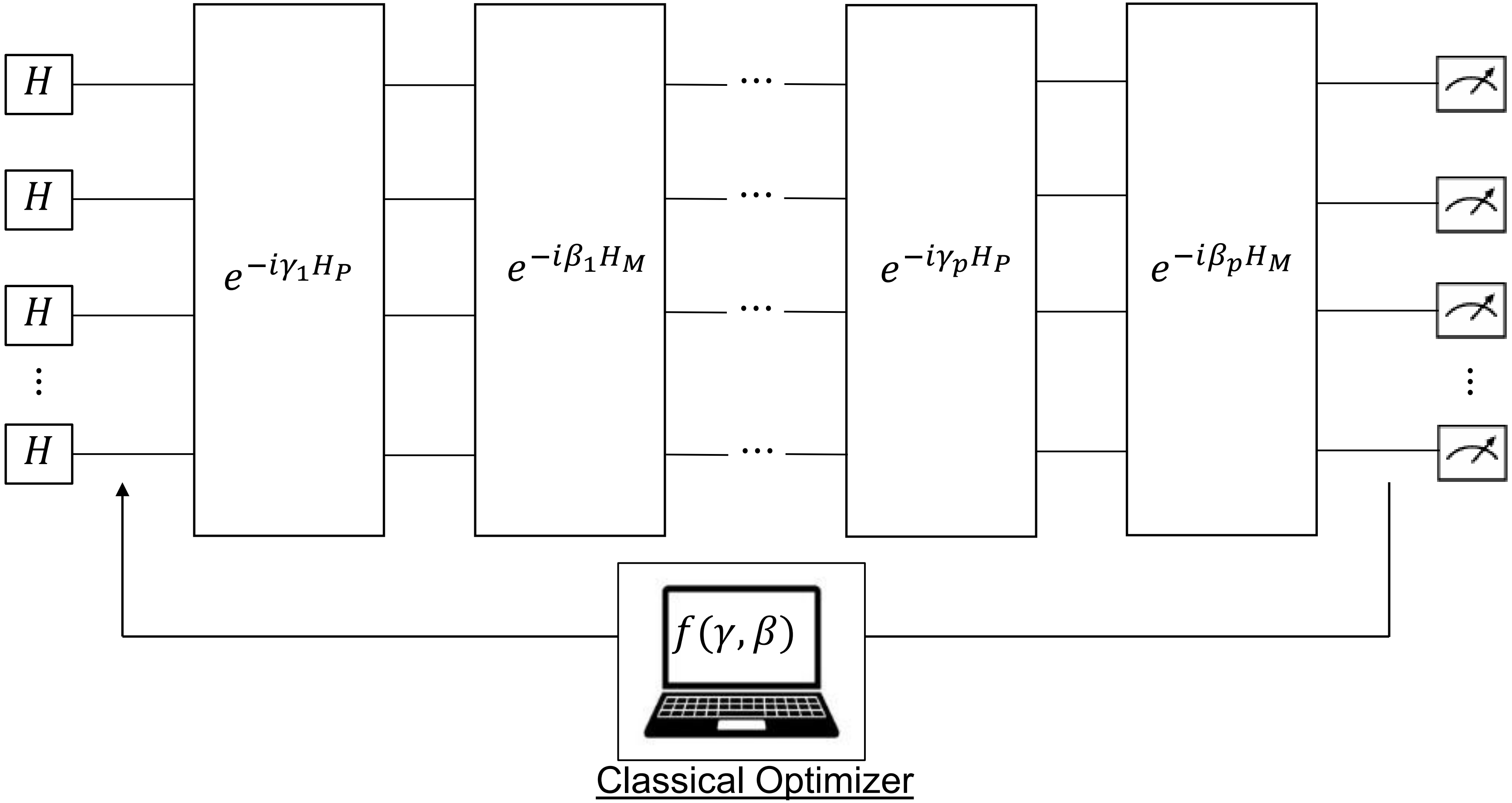
Quantum Anomaly Detection Pipeline



QAOA: Quantum Approximate Optimization Algorithm

Algorithm

1. Instance problem H_P, H_M
2. Initial ansatz on γ, β
3. Run the circuit and measure
4. Classical optimization for $f(\gamma, \beta)$
5. Adjust angles and re-run circuit



E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, "Quantum computation by adiabatic evolution," arXiv preprint quant-ph/0001106, 2000

E. Farhi, J. Goldstone, and S. Gutmann, "A quantum approximate optimization algorithm," arXiv preprint arXiv:1411.4028, 2014

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Algorithm

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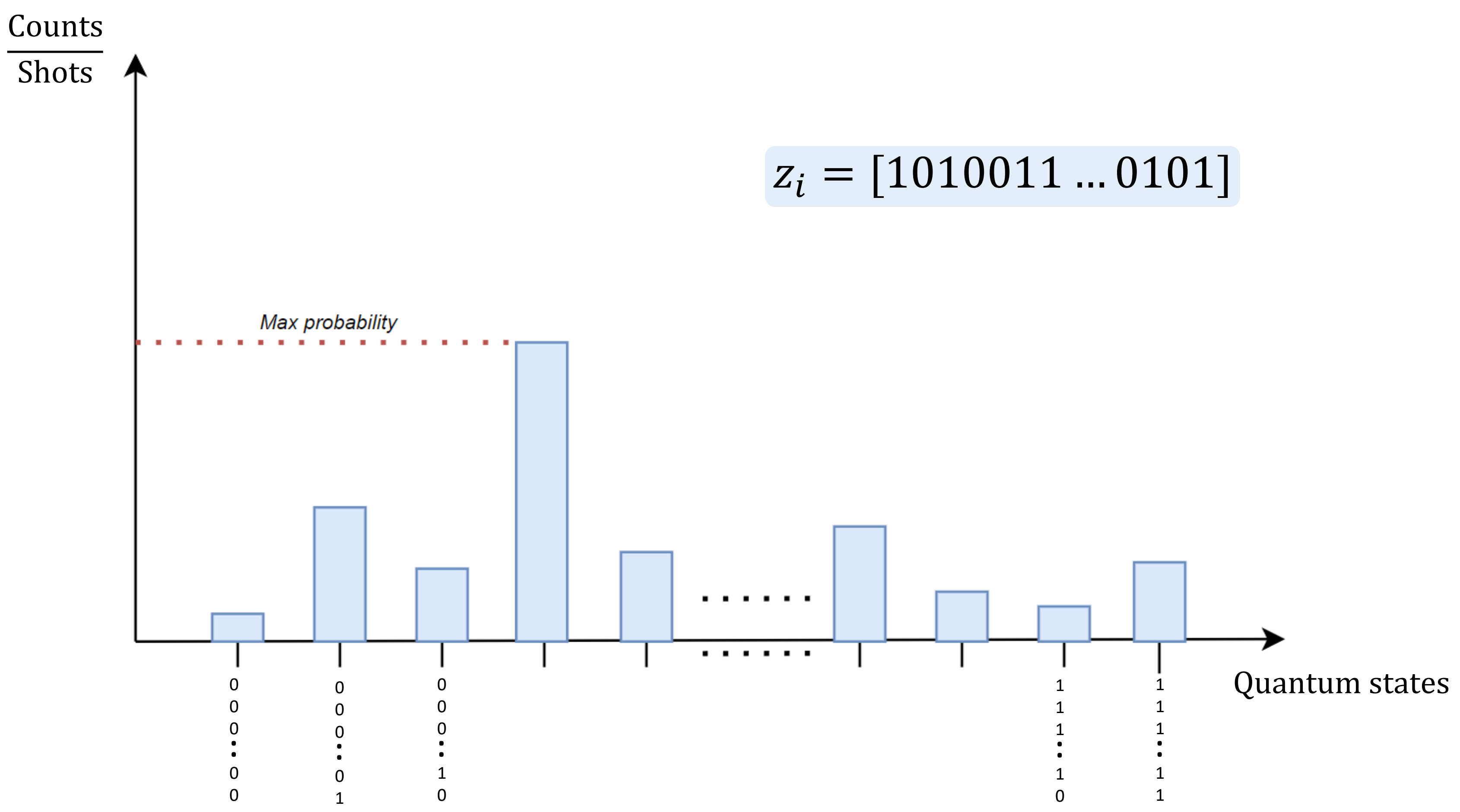
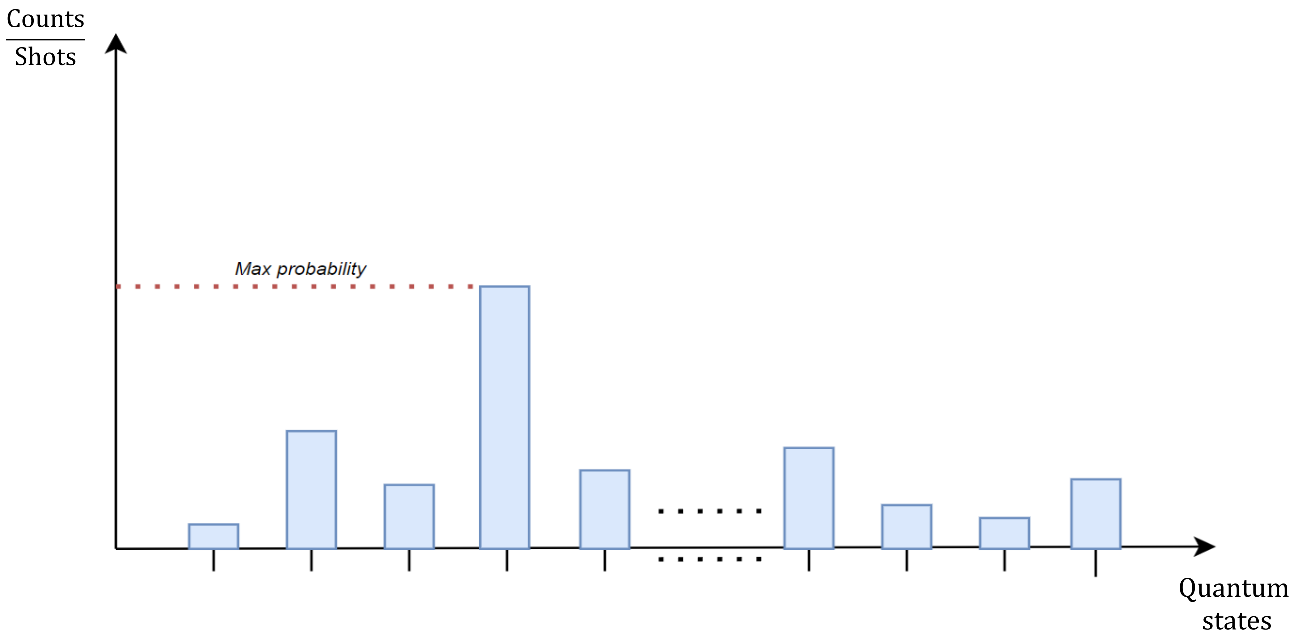
4. Classical optimization for $f(\gamma, \beta)$
5. Adjust angles and re-run circuit
6. Final measurement



Quantum Anomaly Detection Settings

HyperParameters	Parameters
Quantum circuit depth p	Distance metric
Maximum iterations	Fitting model
Mixer H_M	Weights α , β
Classical optimizer	
Initial ansatz START	

Quantum Power



Quantum Solver

$$z_i = [1010011 \dots 0101]$$

$$z_j = [1100100 \dots 0100]$$

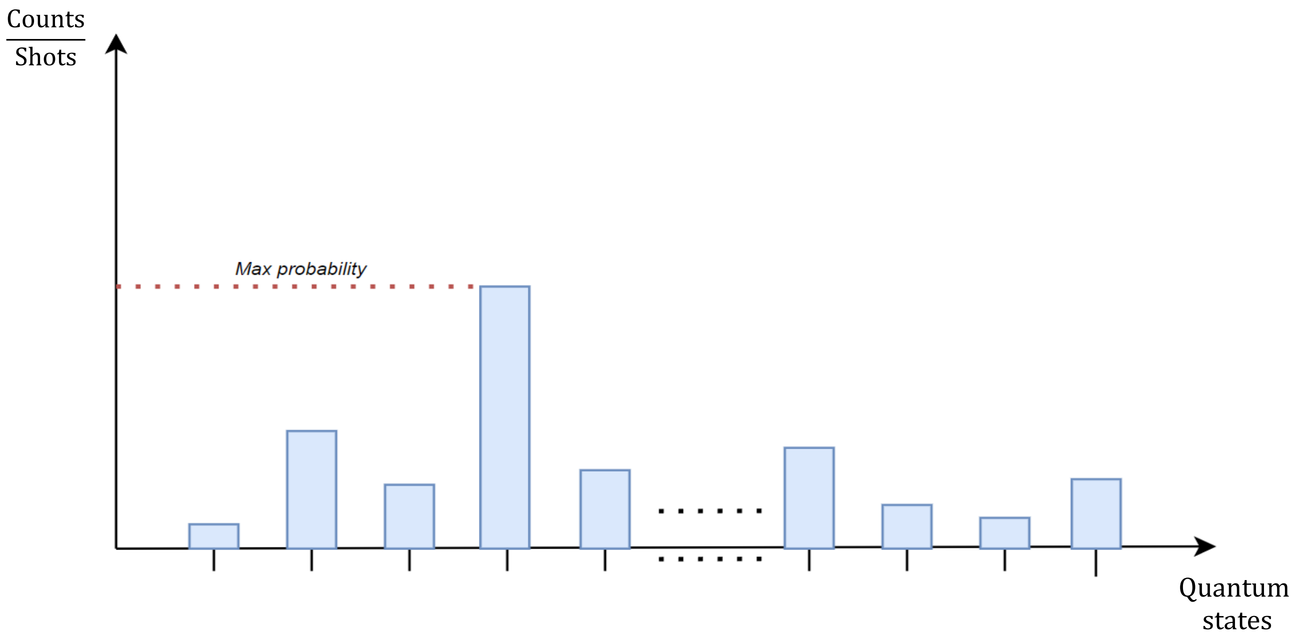
$$\vdots$$

$$z_k^* = [0101010 \dots 1111]$$

Classical Solver

$$z^* = [0101010 \dots 1111]$$

Quantum Power



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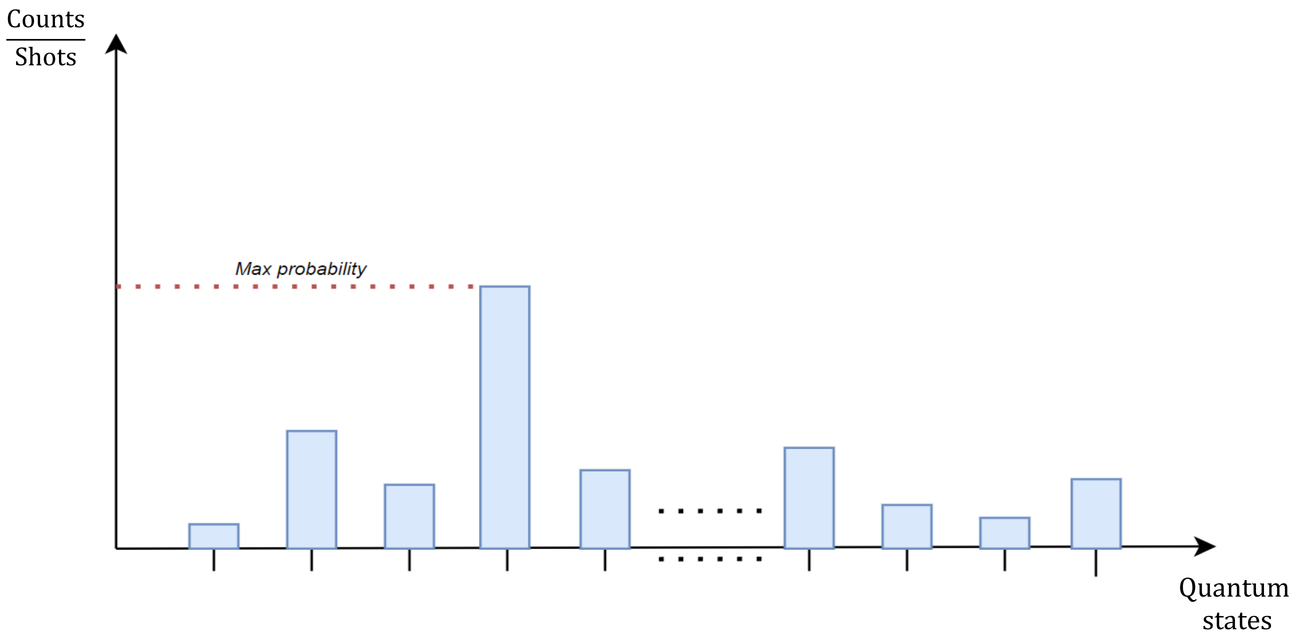

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Anomalies

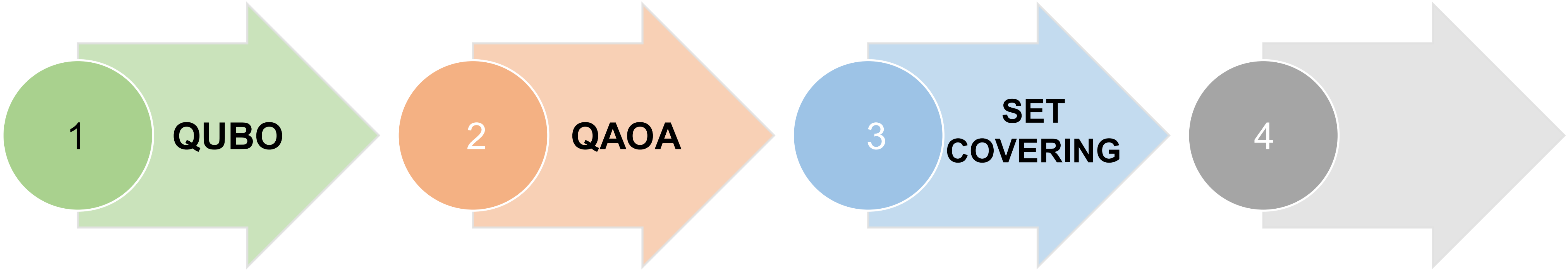
Classical Solver

$$z^* = [0101010 \ldots 1111]$$

PROBLEMs

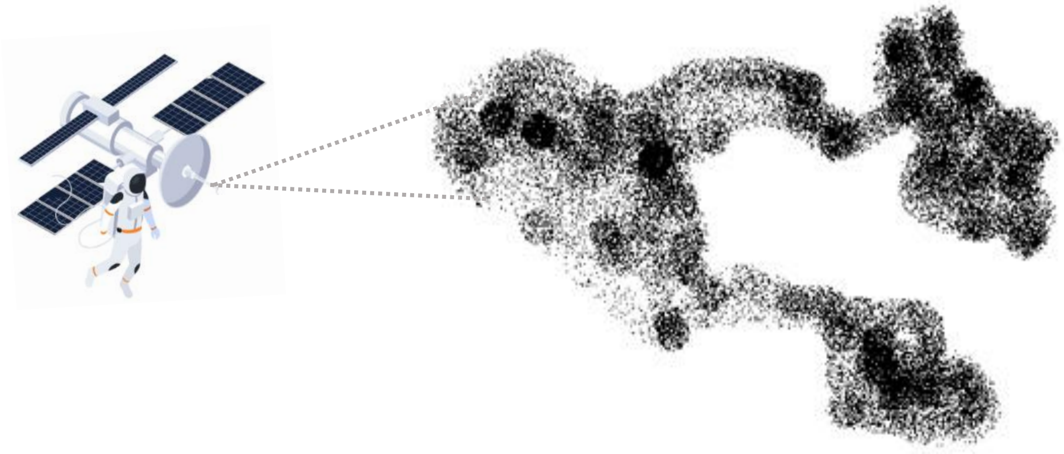
We’re not after a fixed result — we want an architecture that can learn.
The detection acts as a black box — binary outputs with little interpretability.

Quantum Anomaly Detection Pipeline



Set Covering

Inductive Monitoring System (IMS)



Training Data

V. N. Smelyanskiy et al., “A near-term quantum computing approach for hard computational problems in space exploration,” 2012

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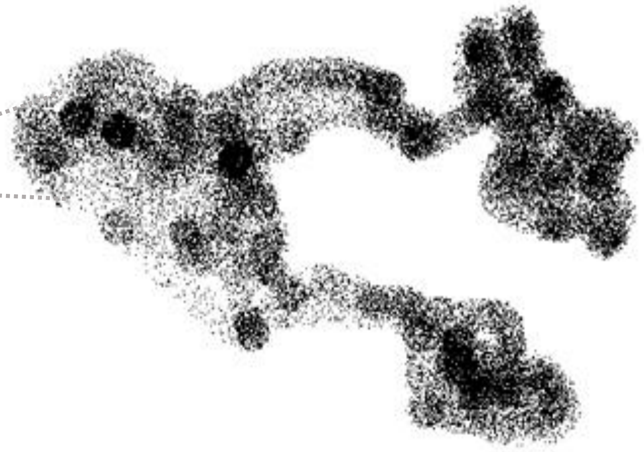

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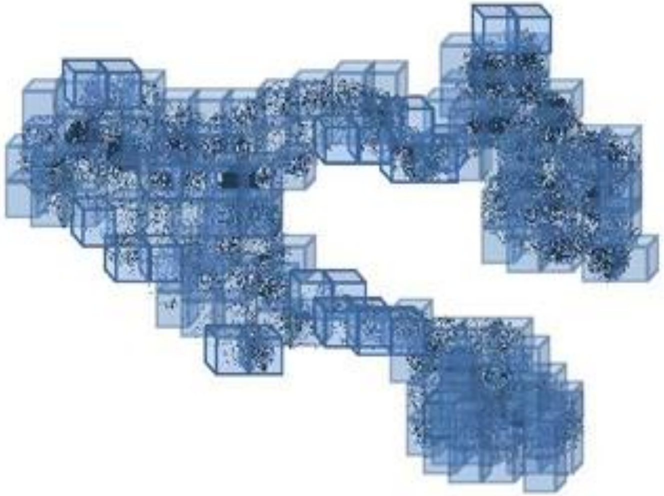

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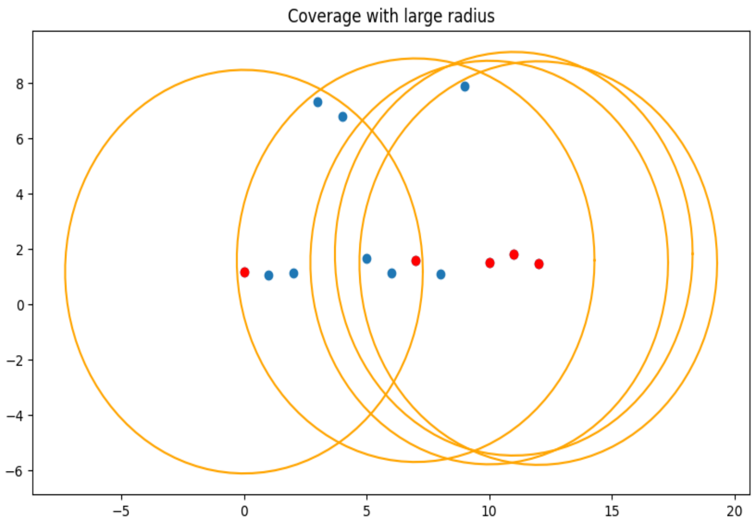
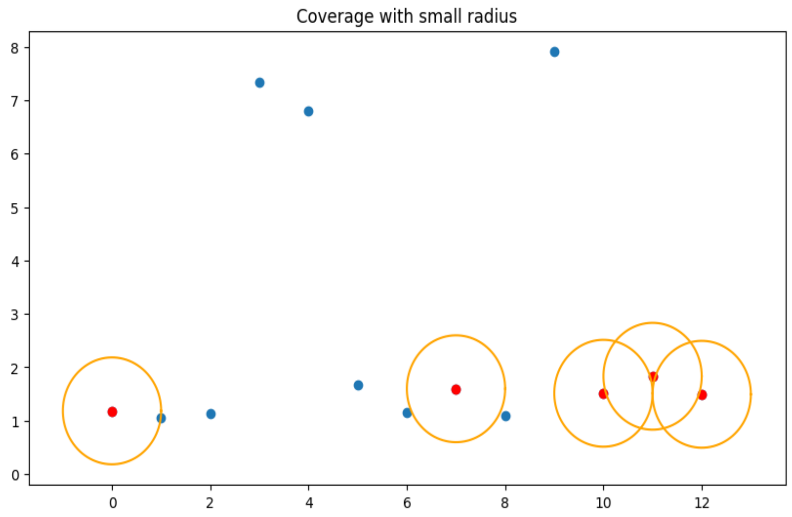
Set Covering

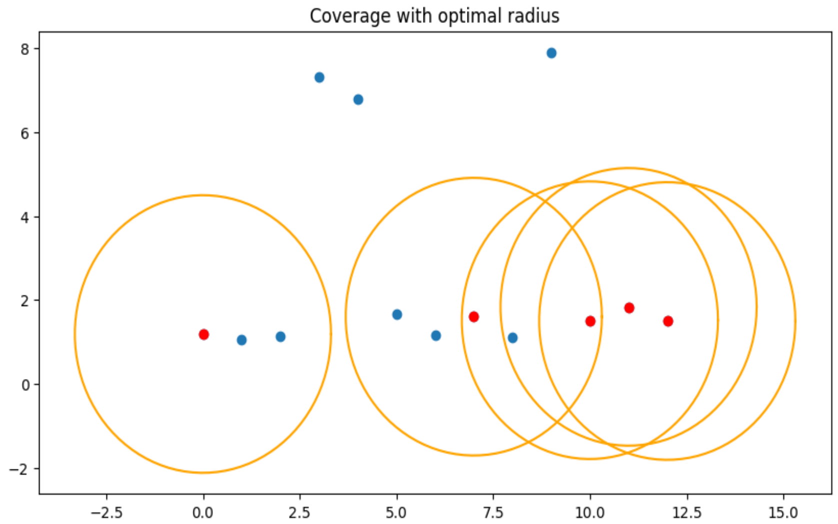
Inductive Monitoring System (IMS)

Quantum Anomaly Detection

$$z_k^* = [0101010 \ldots 0011]$$





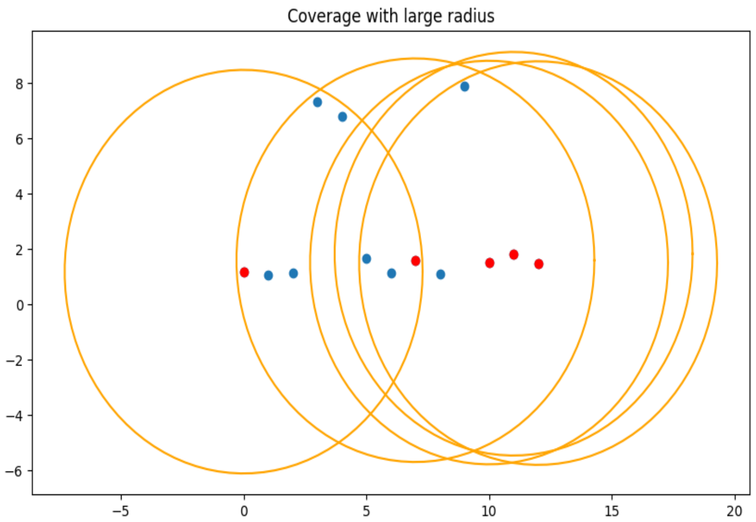


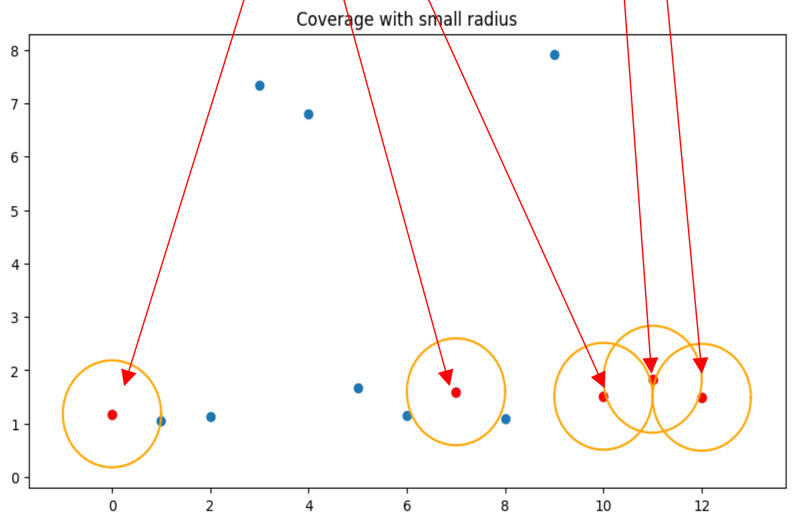
Set Covering

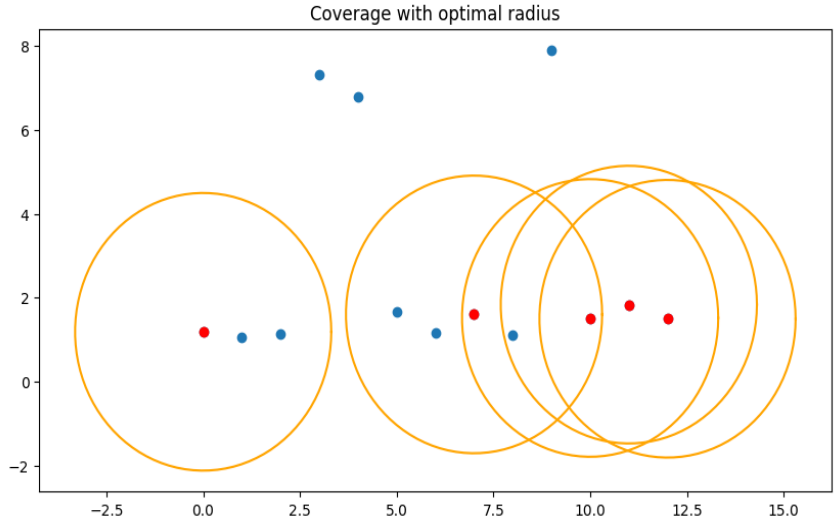
Inductive Monitoring System (IMS)

Quantum Anomaly Detection

$$z_k^* = [0101010\ldots0011]$$ **Centers**







Set Covering

Inductive Monitoring System (IMS)

Quantum Anomaly Detection

$$g_\epsilon(z,\zeta\,,\eta)\;\coloneqq\zeta\sum_{i,j}a_{ij}z_iz_j\;-\eta\sum_i b_iz_i$$

Set Covering

Inductive Monitoring System (IMS)

Quantum Anomaly Detection

$$g_\epsilon(z, \zeta, \eta) := \zeta \sum_{i,j} a_{ij} z_i z_j - \eta \sum_i b_i z_i$$

GOAL

Find ζ^*, η^* $\xrightarrow{\text{such that}}$ Given z^*
MAX normal points within the covering
MAX anomalies outside the covering

Set Covering

Inductive Monitoring System (IMS)

Quantum Anomaly Detection

$$g_\epsilon(z,\zeta,\eta) \coloneqq \zeta \sum_{i,j} a_{ij} z_i z_j - \eta \sum_i b_i z_i$$

GOAL

Find ζ^*, η^* $\xrightarrow{\text{such that}}$ Given z^*
MAX normal points within the covering
MAX anomalies outside the covering

- ϵ represents the covering;
- a_{ij} penalizes the overlaps;
- b_i encourages all points to be included in at least one box;
- $\zeta,\ \eta$ tunable parameters.

Quantum Anomaly Detection Settings

HyperParameters		Parameters
Quantum circuit depth	p	Distance metric
Maximum iterations		Fitting model
Mixer	H_M	Weights α , β
Classical optimizer		
Initial ansatz	START	

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Classical optimizer		Covering
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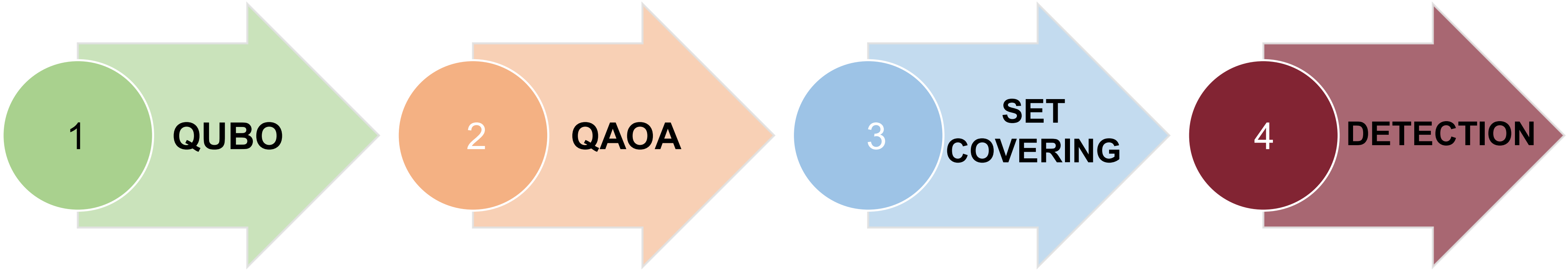
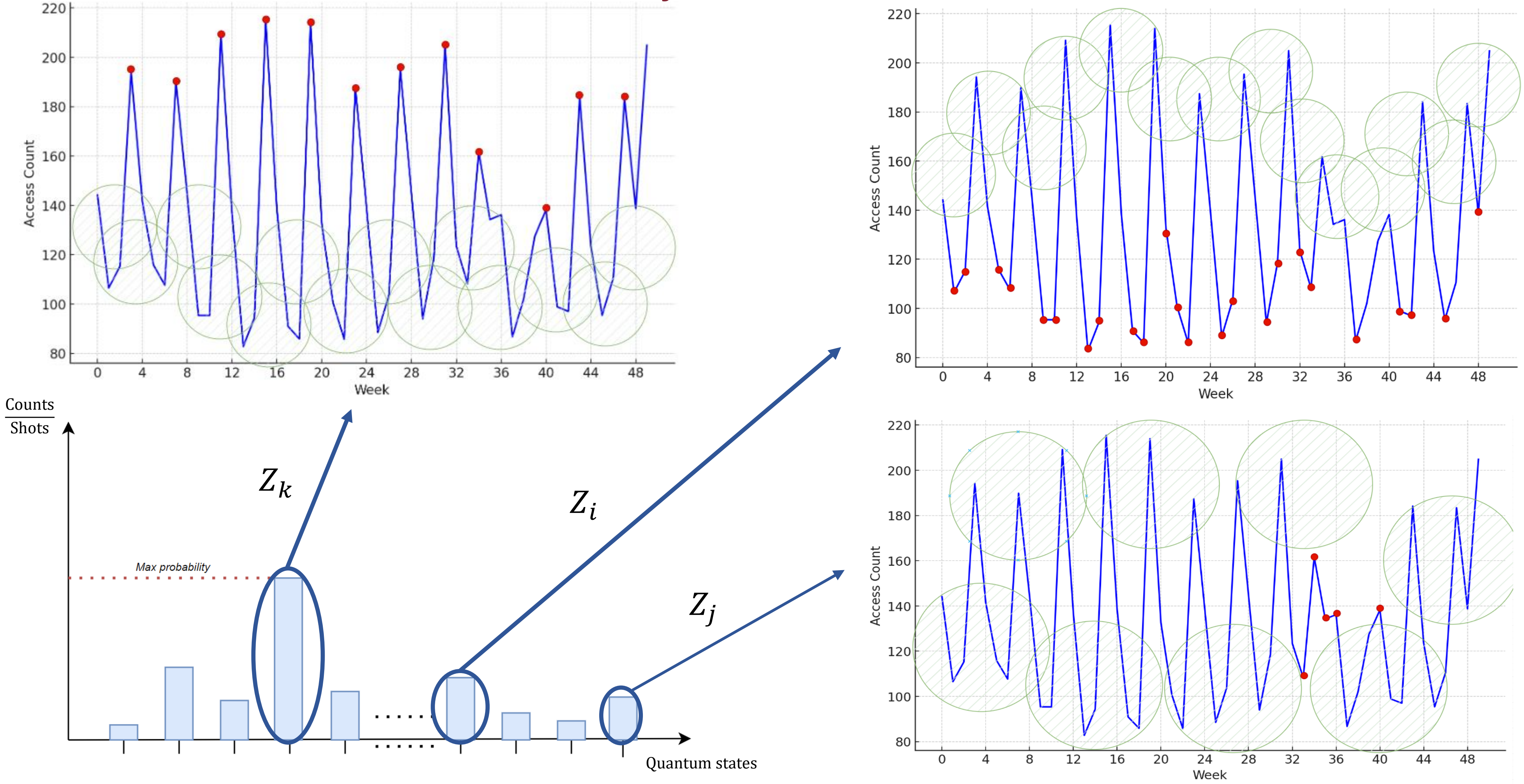


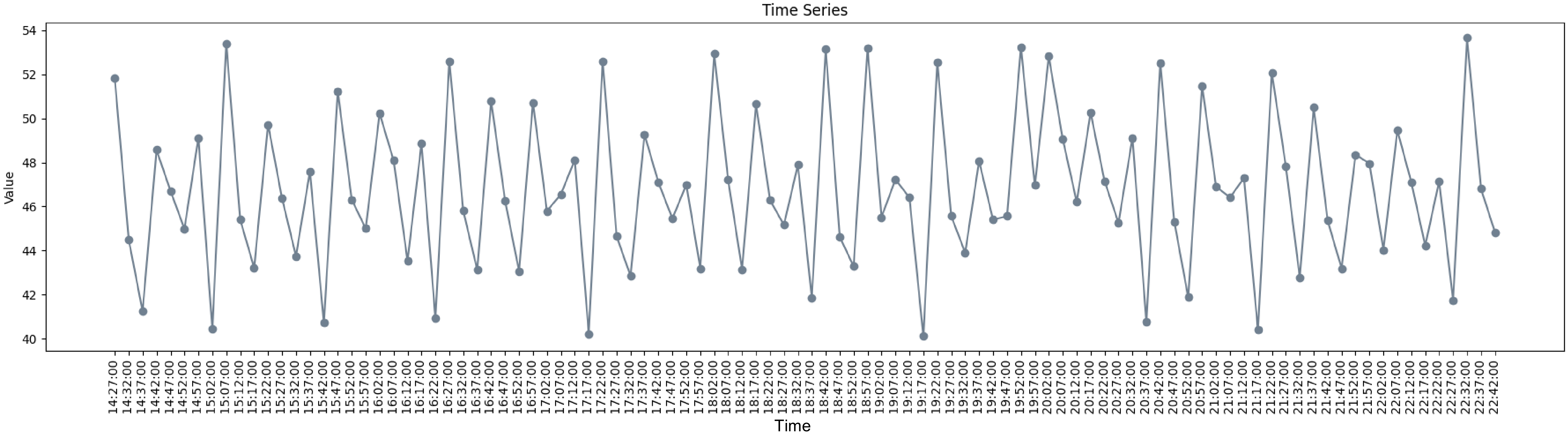
Experiments & Results

Weekly website access count



Experiments & Results

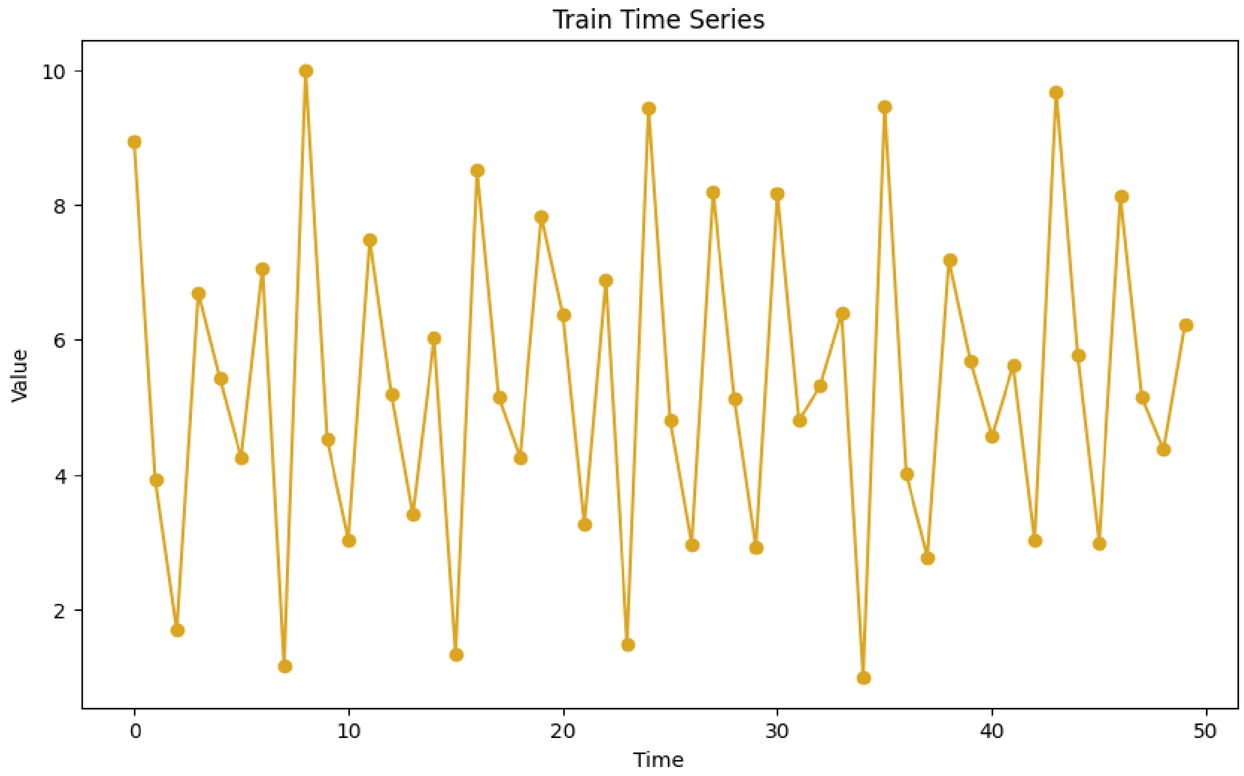
Daily CPU utilization

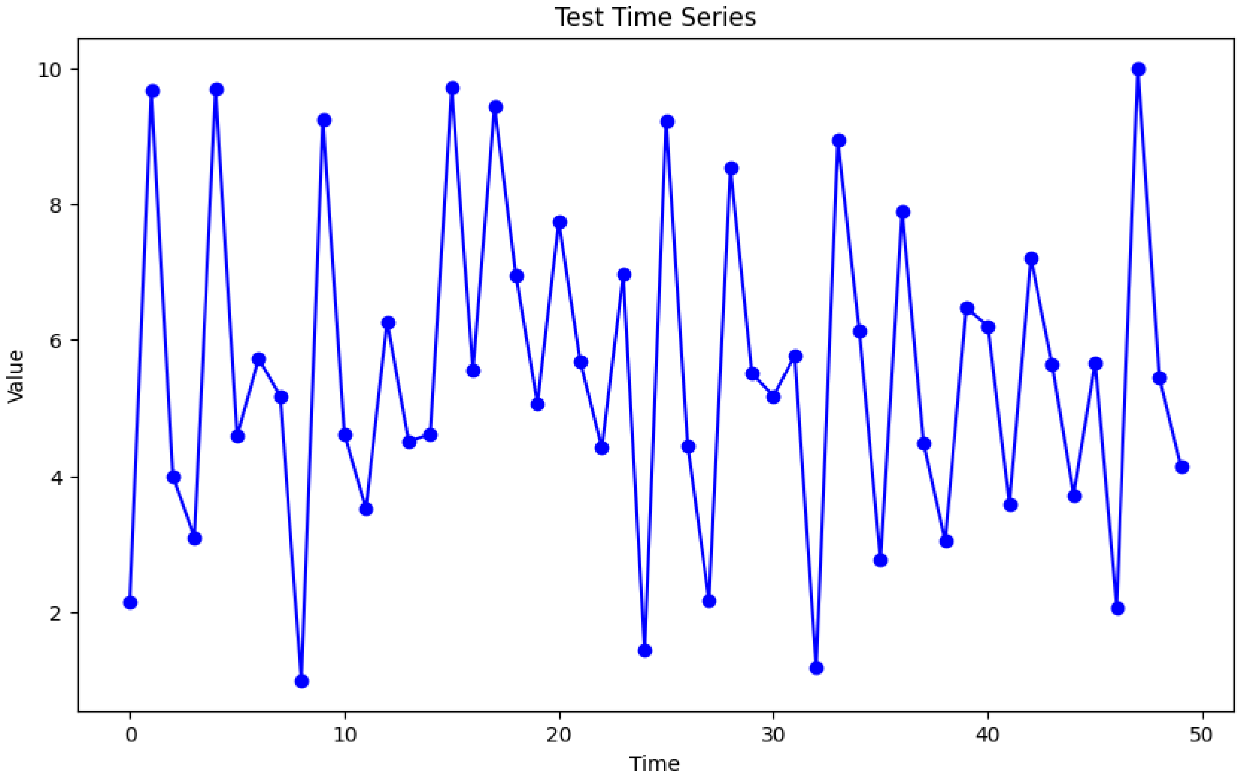


Numenta Anomaly Benchmark (NAB) dataset: $ec2\_cpu\_utilization\_5f5533$

Experiments & Results

Daily CPU utilization





Numenta

Experiments & Results

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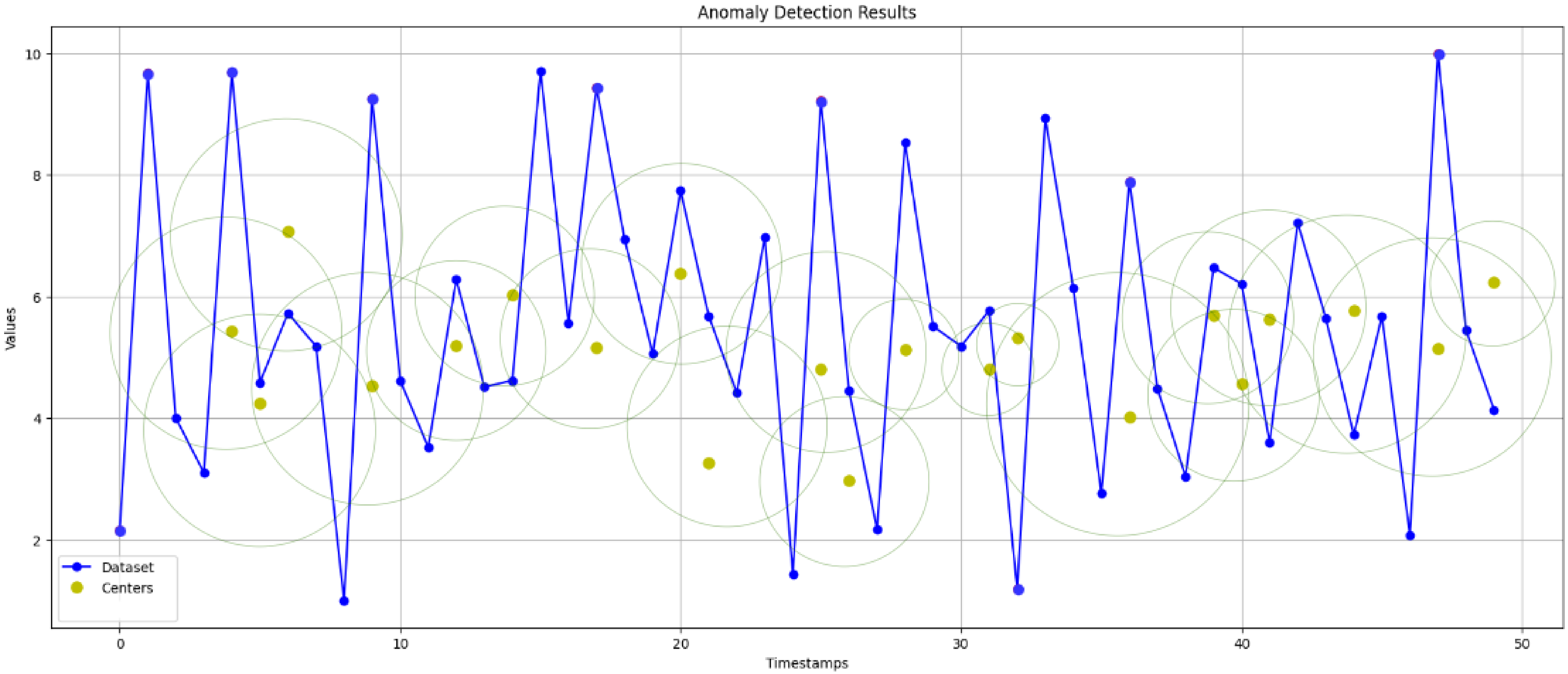

Experiments & Results

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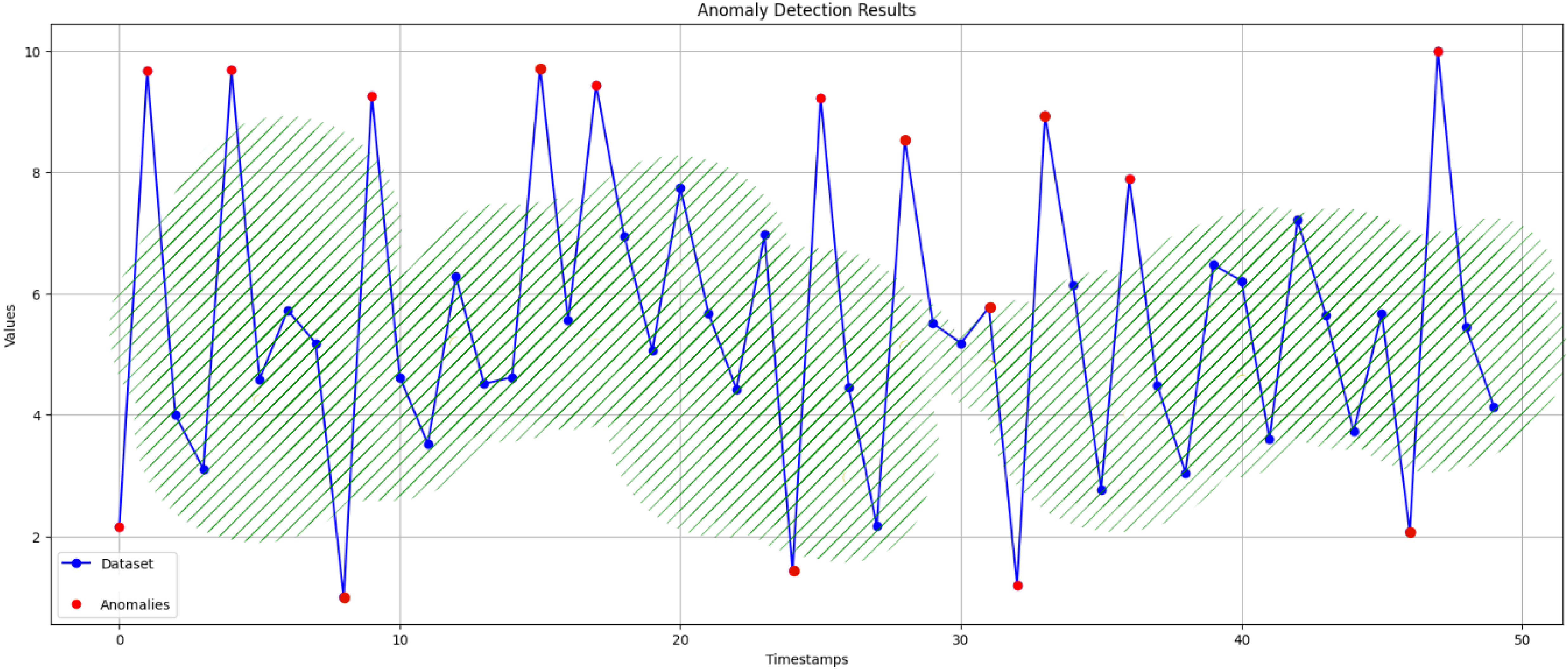

Experiments & Results

	Precision	Time
QAD	85.00%	$\sim 10\ s$
Isolation Forest	80.00%	$\sim 1\ s$
Local Outlier Factor	100.00%	$\sim 2\ s$
DBSCAN	100.00%	$\sim 1\ s$
ONE-Class SVM	70.00%	$\sim 2\ s$

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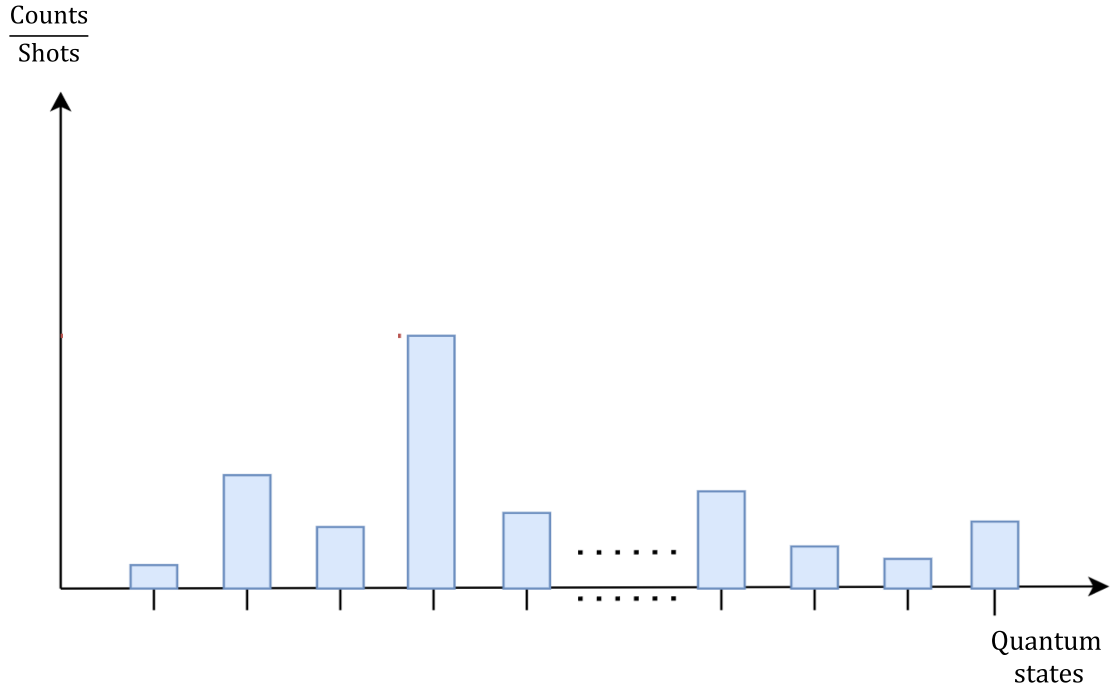

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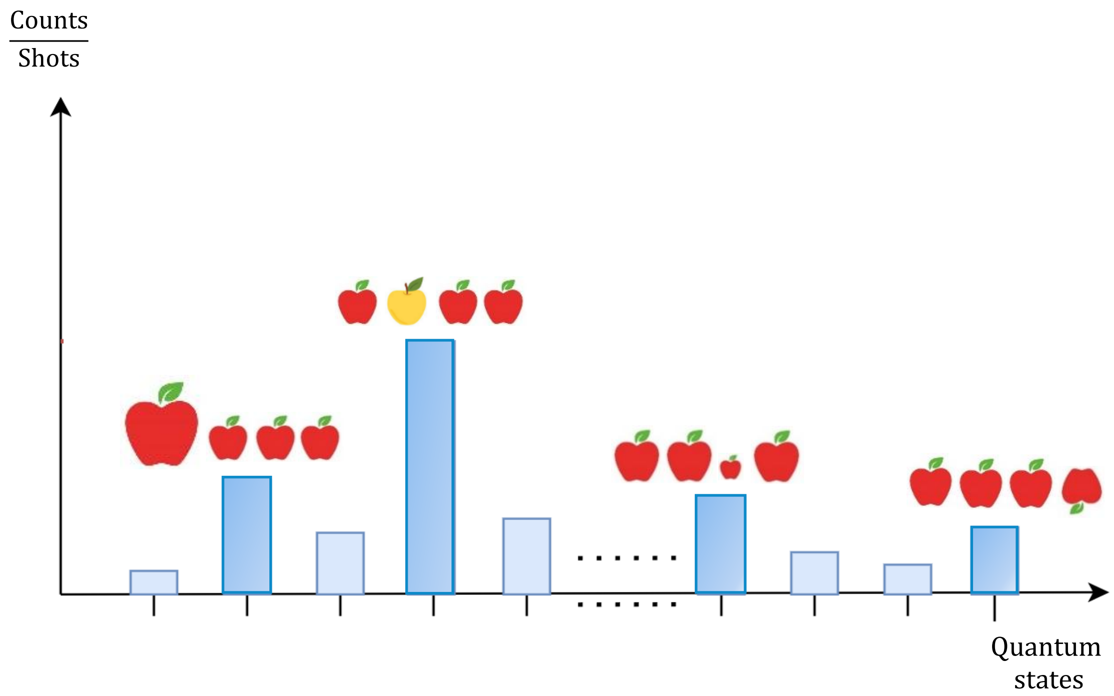

Numenta

Experiments & Results

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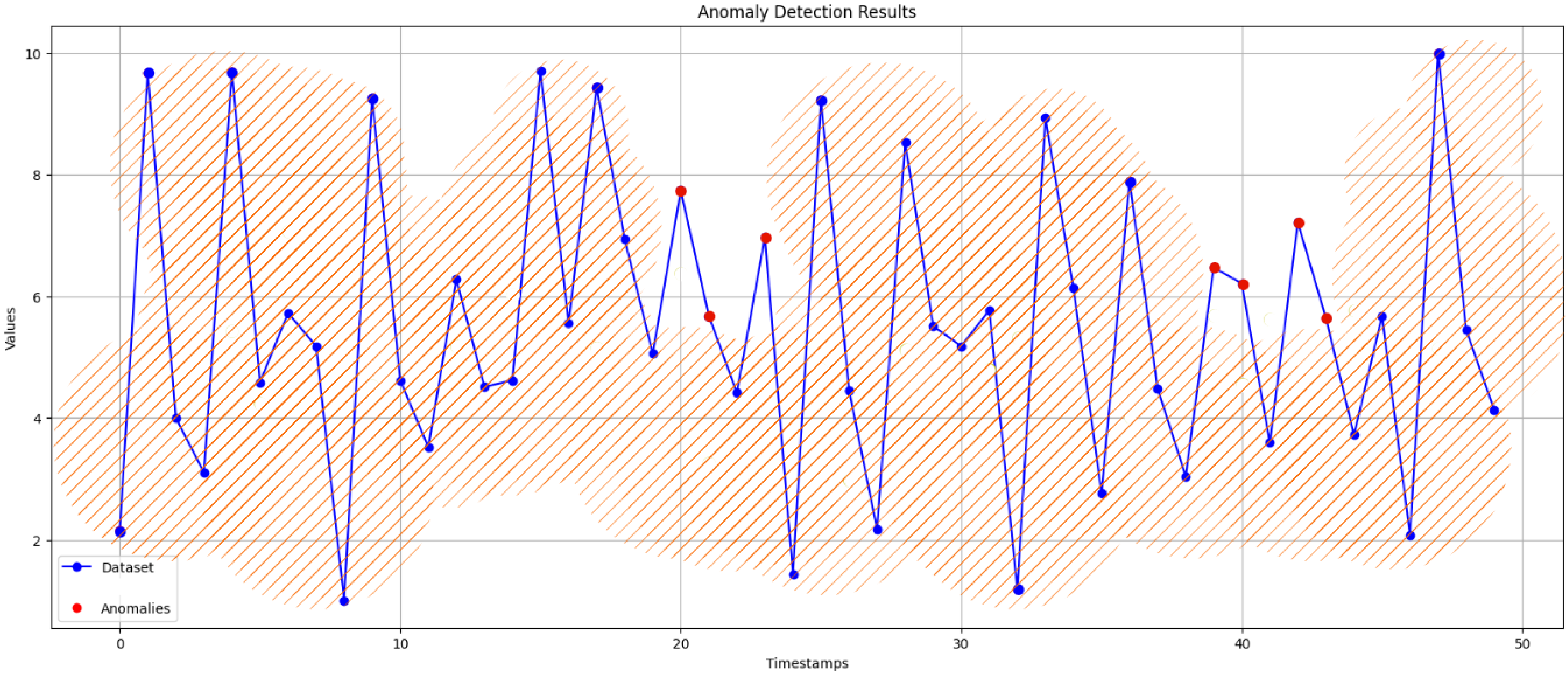

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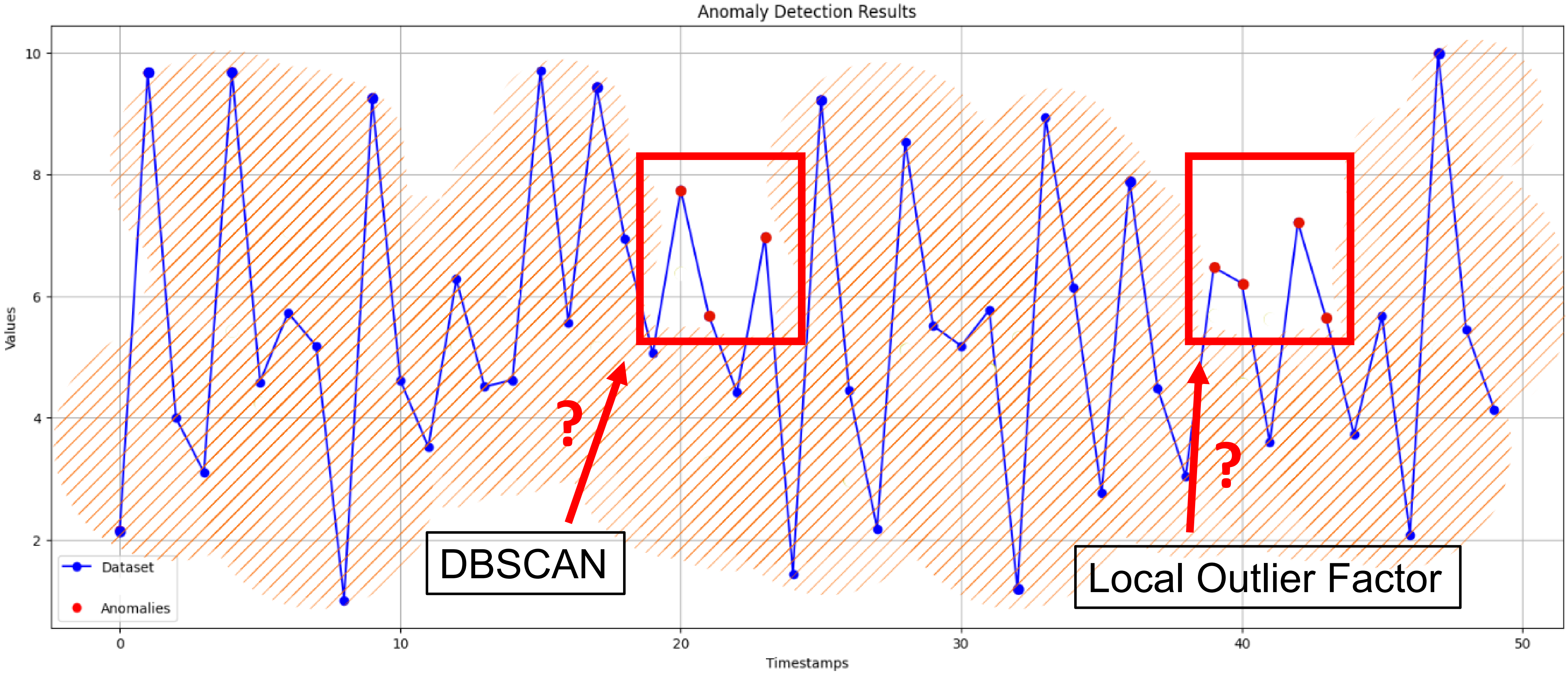

What did we achieve

- Customizable detection
- High interpretability
- General-purpose efficiency
- Novel approach

What's up next

- Model and metric selection
- Long-range scalability
- Multivariate detection

NESYA LABORATORY

DIET

Dipartimento di Ingegneria dell'Informazione, Elettronica e Telecomunicazioni

Thank you for Your attention!

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