

# Novel Quantum Approaches to Hyperdimensional Computing for Neural Networks

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# Motivation & Context

- Hyper Dimensional Computing (**HDC**), is a brain-inspired computing paradigm that models cognitive processes through high-dimensional distributed representations called **hypervectors**



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- Hyper Dimensional Computing (HDC), is a brain-inspired computing paradigm that models cognitive processes through high-dimensional distributed representations called hypervectors
- The **role of hypervectors** in HDC can be interpreted as **distributed activation patterns** across an abstract set of neurons, in which each bit represents the activation state at a particular time so we can associate to **HDC architectures** a role analogue to **Neural Networks**





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- The role of hypervectors in HDC can be interpreted as distributed activation patterns across an abstract set of neurons, in which each bit represents the activation state at a particular time so we can associate to HDC architectures a role analogue to Neural Networks
- **HDC acts through** a set of well-defined **operations** that enable the encoding, manipulation, and comparison of data, and these operations can be **defined with different approaches**.



# Motivation & Context

- Despite its promise, HDC's reliance on specialized hardware imposes **inherent limitations**



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- Despite its promise, HDC's reliance on specialized hardware imposes inherent limitations
- **Quantum computing**, which leverage principles such as entanglement and superposition, enable the management of information in fundamentally distinct ways from classical systems, yielding **remarkable speedups**



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- Quantum computing, which leverage principles such as entanglement and superposition, enable the management of information in fundamentally distinct ways from classical systems, yielding remarkable speedups
-  **Quantize** the components of **HDC architectures!**



Mmmh.... How...



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# Methodology

-  Stop making cat memes



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- Quantize one element at a time: first the associative memory (i.e. a dictionary of hypervectors), then the operations
- **Theorem 1 (informal):** For every HDC architecture it is possible to quantize both the associative memory and the operations, but
  - If you **start by quantizing the memory** you **automatically get quantized operations** (e.g. binding)
  - If you **start by quantizing the operations** you can **keep the memory classical**



# Example

- I'll **show** you only the **first process**...



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# Example

- I'll **show** you only the **first process**...
- There **exists a quantum version of the associative memory** (Martinez & Ventura ~2000) that **uses** only the **superposition principle**
- You start by encoding each hypervector in the dictionary into a **quantum state**, by "**carving out one amplitude at a time**".

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$



# Example

- Use **Grover's algorithm** (Grover ~1996) to perform **inference and retrieval**

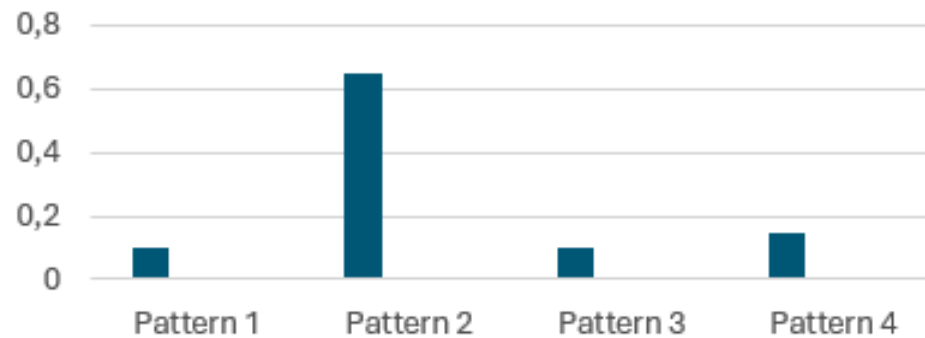
$$U_{z^*} := \begin{cases} -|z\rangle & \text{if } z = z^* \\ |z\rangle & \text{otherwise} \end{cases} \rightarrow U_{g(f(\cdot))} := \begin{cases} -|z\rangle & \text{if } z = f(\cdot) \\ |z\rangle & \text{otherwise} \end{cases}$$



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[github.com/leonardoLavagna/qhdc](https://github.com/leonardoLavagna/qhdc)



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# Some results

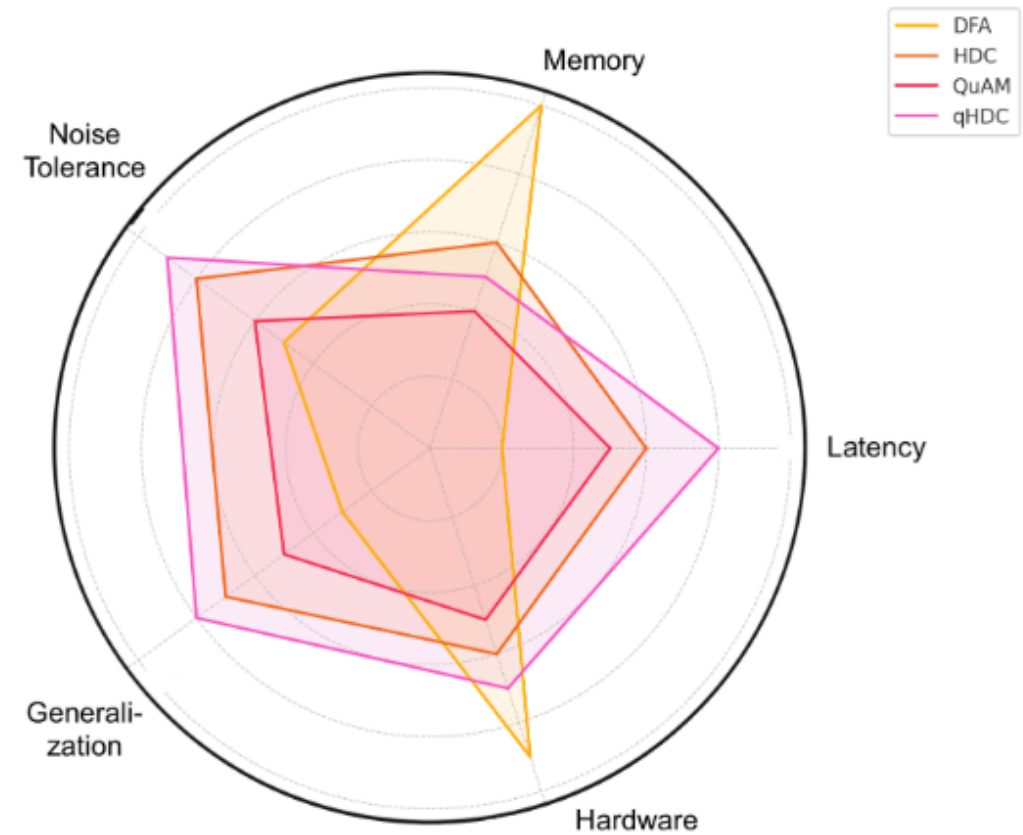
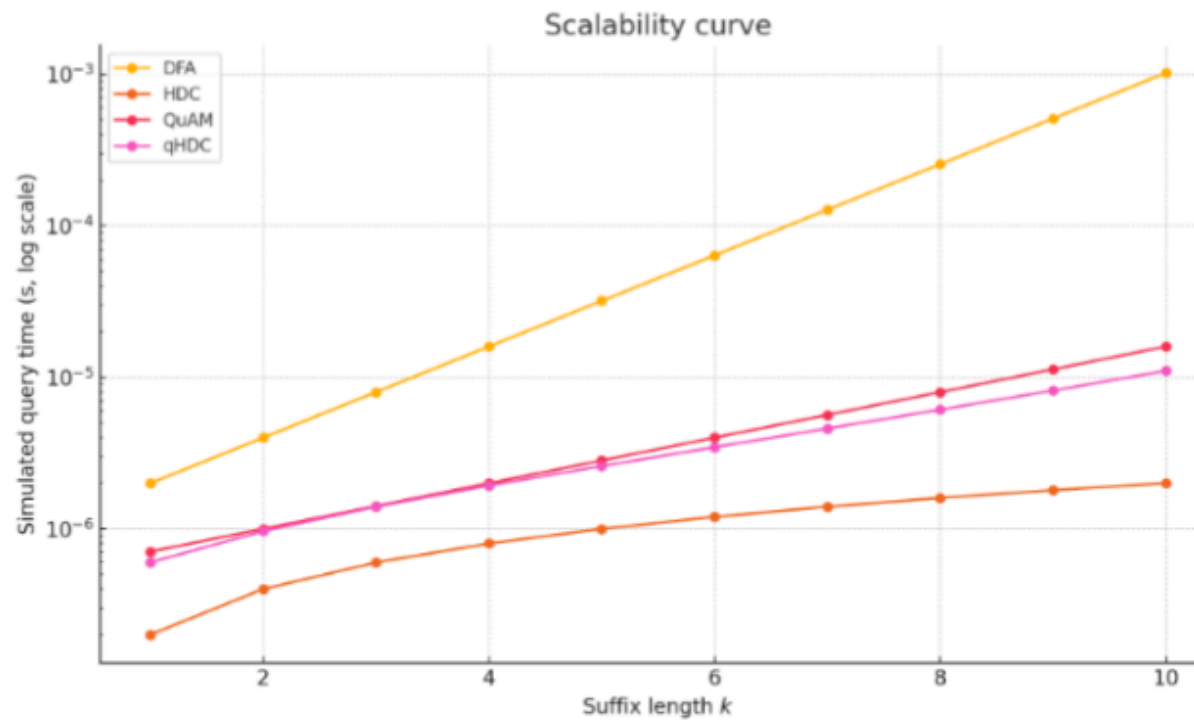
- Consider the **pattern completion problem** summarized below

Let  $y \in \{0, 1\}^{n-k}$  be a partially observed binary sequence of length  $n - k$ , and let  $\omega \in \{0, 1\}^k$  be an unknown binary suffix of length  $k$ . The goal is to find a completed sequence  $z = y\omega \in \{0, 1\}^n$  that satisfies a set of constraints  $C$ .

# Some results

- Consider the pattern completion problem
- The proposed quantum-HDC architectures
  - show **better scaling**
  - have a **balanced performance** in terms of noise tolerance, memory requirements, latency and generalization capabilities





# Conclusions

- **We can do it** and there can be some advantages (e.g. an exponential gain in memory size) always subject to tradeoffs (e.g. a slowdown in pattern retrieval)



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- We don't have good enough **quantum hardware**



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- There are "deep" reasons that compell you to **limit the applications to "pattern-completion-type" problems**



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- **Theorem 2 (informal):** we can **solve a machine learning problem** with a **quantum HDC architecture** if and only if we can **map it to a pattern completion problem** (cf., regular languages and finite automata)



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